Materials Selection for Mechanical Design II

A Brief Overview of a Systematic Methodology

Material and Shape Selection
Method for Early Technology Screening

- Design performance is determined by the combination of:
  - Shape
  - Materials
  - Process

- Underlying principles of selection are unchanged
  - BUT, do not underestimate impact of shape or the limitation of process
Material and Shape Selection

- Performance isn't just about materials - shape can also play an important role
- Shape can be optimized to maximize performance for a given loading condition
- Simple cross-sectional geometries are not always optimal
  - Efficient Shapes like I-beams, tubes can be better
- Shape is limited by material
  - Wood can be formed only so thin
- Goal is to optimize both shape and material for a given loading condition
Loading Conditions and Shape

- Different loading conditions are enhanced by maximizing different geometric properties
- Area for tension
- Second moment for compression and bending
- Polar moment for torsion

Figure by MIT OCW.
Shapes and Moments

<table>
<thead>
<tr>
<th>Shape</th>
<th>Area</th>
<th>Second Moment</th>
<th>Polar Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>$bh$</td>
<td>$\pi r^2$</td>
<td>$\frac{bh^3}{12}$</td>
</tr>
<tr>
<td>Circle</td>
<td>$\pi r^2$</td>
<td>$\frac{\pi}{4} r^4$</td>
<td>$\frac{\pi}{4} (r_o^4 - r_i^4)$</td>
</tr>
<tr>
<td>Tube</td>
<td>$2t(h+b)$</td>
<td>$\frac{1}{6} h^3 t \left(1 + \frac{3b}{h}\right)$</td>
<td>$\frac{1}{6} h^3 t \left(1 + \frac{3b}{h}\right)$</td>
</tr>
<tr>
<td>Square Tube</td>
<td>$2t(h+b)$</td>
<td>$\frac{2tb^2 h^2}{(h+b)} \left(1 - \frac{t}{h}\right)^4$</td>
<td>$\frac{2}{3} b t^3 \left(1 + \frac{4h}{b}\right)$</td>
</tr>
</tbody>
</table>
Shape Factor Definition

- Shape factor measures efficiency for a mode of loading given an equivalent cross-section
  - “Efficiency”: For a given loading condition, section uses as little material as possible
- Defined as 1 for a solid cross-section
  - Higher number is better, more efficient

\[
\phi^e = \frac{S}{S_o}
\]

For elastic cases:
\(\phi = \) shape factor
\(S = \) stiffness of cross-section under question
\(S_o = \) stiffness of reference solid cross-section
Shape Factor for Elastic Bending

\[ \phi_B^e = \frac{S}{S_o} = \frac{EI}{EI_o} = \frac{I}{I_o} \]

Reference solid cross-section

\[ I_o = \frac{b_o^4}{12} = \frac{A_o^2}{12} \]

Compare sections of same area \( \Rightarrow \)

\[ A_o = A \]

Notice that shape factor is dimensionless
I-Beam Elastic Bending Shape Factor

For these dimensions, the shape increased stiffness over 13 times while using the same amount of material!

Is this design possible in all materials?
Materials Limit Best Achievable Shape Factor

- Shape efficiency dependent on material
- Constraints: manufacturing, material properties, local buckling
  - For example, can’t have thin sections of wood
- Values in table determined empirically
- Note: previous design not possible in polymers, wood \((\phi_B^e)=13.5\)

<table>
<thead>
<tr>
<th>Material</th>
<th>((\phi_B^e)_{\text{max}})</th>
<th>((\phi_T^e)_{\text{max}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structural Steels</td>
<td>65</td>
<td>25</td>
</tr>
<tr>
<td>Aluminum Alloys</td>
<td>44</td>
<td>31</td>
</tr>
<tr>
<td>GFRP and CFRP</td>
<td>39</td>
<td>26</td>
</tr>
<tr>
<td>Polymers</td>
<td>12</td>
<td>8</td>
</tr>
<tr>
<td>Woods</td>
<td>6</td>
<td>1</td>
</tr>
</tbody>
</table>
Shape Factors and Material Indices

Example: Bending Beam

Mass: \( m = AL\rho \)

Bending Stiffness: \( S = \frac{F}{\delta} \geq \frac{CEI}{L^3} \)

Shape Factor: \( \phi_B^e = \frac{I}{I_o} = \frac{12I}{A^2} \)

Replace \( I \) in Stiffness using \( \phi_B^e \): \( S = \frac{C}{12} \frac{E}{L^3} \phi_B^e A^2 \)

Eliminate \( A \) from mass using stiffness: \( m = \left( \frac{12S}{C} \right)^{1/2} L^{5/2} \left[ \frac{\rho}{\left( \phi_B^e E \right)^{1/2}} \right] \)

Material Index: \( M = \frac{\left( \phi_B^e E \right)^{1/2}}{\rho} \)  

Previously: \( M = \frac{E^{1/2}}{\rho} \)
**Shape Factors and Material Indices: Beams**

**Objective:** Minimize Mass

**Performance Metric:** Mass

<table>
<thead>
<tr>
<th>Loading</th>
<th>Stiffness Limited</th>
<th>Strength Limited</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension</td>
<td>$\frac{E}{\rho}$</td>
<td>$\frac{\sigma_f}{\rho}$</td>
</tr>
<tr>
<td>Bending</td>
<td>$(\phi^e_B E)^{1/2}/\rho$</td>
<td>$(\phi^f_B \sigma_f)^{2/3}/\rho$</td>
</tr>
<tr>
<td>Torsion</td>
<td>$(\phi^e_T G)^{1/2}/\rho$</td>
<td>$(\phi^f_T \sigma_f)^{2/3}/\rho$</td>
</tr>
</tbody>
</table>

Maximize!
Shape Factors Affect Material Choice

- Shape factors can dramatically improve performance for a given loading condition.
- The optimal combination of shape and material leads to the best design.

Diagram: A graph showing the relationship between modulus (GPa) and density (Mg/m³) for various materials, including Elastomers, Polymers, Woods, Composites, Ceramics, Metals, and Foams. The diagram includes lines labeled $M_{\phi=1}$ and $M_{\phi=10}$ to illustrate the effect of shape factors on material performance.
Example Problem: Bicycle Forks

Bicycle forks need to be lightweight
- Primary constraint can be stiffness or strength
- Toughness and cost can be other constraints

Photos of bicycle forks removed for copyright reasons.
Bicycle Forks: Problem Definition

- **Function:**
  - Forks - support bending loads

- **Objective:**
  - Minimize mass

- **Constraints:**
  - Length specified
  - Must not fail (strength constraint)

- **Free variables:**
  - Material
  - Area: Tube radius OR thickness OR shape

**Objective:**
\[ m = AL\rho \]

**Constraint:**
\[ \sigma = \frac{M_{ym}}{I} = \frac{FL_{ym}}{I} \leq \sigma_f \]

**Free Variables:**

**Solid Tube:**
\[ A = \pi r^2 \quad I = \frac{\pi r^4}{4} \]

**Hollow Tube:**
\[ A \approx 2\pi rt \quad I \approx \pi r^3 t \]

**Shape:**
\[ \phi_B^f = \frac{4\sqrt{\pi Z}}{A^{3/2}} \quad Z = \frac{I}{y_m} \]
**Material Indices: Shape specified**

**Free variable definition important**

### Solid Section

**Free Variable: Area**

\[ \sigma = \frac{My_m}{I} \leq \sigma_f \]

\[ \sigma_f \geq \frac{4FL}{\pi r^3} \]

**Solve for** \( r \):

\[ r = \left( \frac{4FL}{\pi \sigma_f} \right)^{1/3} \]

**Substitute into** \( m \):

\[ m = \pi^{1/3} \left( 4F \right)^{2/3} L^{2/3} \left[ \frac{\rho}{\sigma_f^{2/3}} \right] \]

**Maximize:** \( M = \left[ \frac{\sigma_f^{2/3}}{\rho} \right] \)

### Hollow Section

**Free Variable: Radius**

\[ \sigma = \frac{My_m}{I} \leq \sigma_f \]

\[ \sigma_f \geq \frac{FL}{\pi r^2 t} \]

**Solve for** \( r \):

\[ r = \left( \frac{FL}{\pi t \sigma_f} \right)^{1/2} \]

**Substitute into** \( m \):

\[ m = \left( 4\pi F \right)^{1/2} \left( L^2 t \right)^{1/2} \left[ \frac{\rho}{\sigma_f^{1/2}} \right] \]

**Maximize:** \( M = \left[ \frac{\sigma_f^{1/2}}{\rho} \right] \)

### Hollow Section

**Free Variable: Thickness**

\[ \sigma = \frac{My_m}{I} \leq \sigma_f \]

\[ \sigma_f \geq \frac{FL}{\pi r^2 t} \]

**Solve for** \( t \):

\[ t = \frac{FL}{\pi r^2 \sigma_f} \]

**Substitute into** \( m \):

\[ m = 2F \left( \frac{L^2}{r} \right) \left[ \frac{\rho}{\sigma_f} \right] \]

**Maximize:** \( M = \left[ \frac{\sigma_f}{\rho} \right] \)
**Material Index with Shape Free**

$$\sigma_f \geq \frac{F L y_m}{I} = \frac{F L}{Z} = \frac{F L 4\sqrt{\pi}}{\phi_B A^{3/2}}$$

**Solve for** $A$:

$$A = \left(\frac{F L 4\sqrt{\pi}}{\phi_B \sigma_f}\right)^{2/3}$$

**Substitute into** $m$:

$$m = \left(4\sqrt{\pi} F\right)^{2/3} L^{5/3} \frac{\rho}{\left(\phi_B \sigma_f\right)^{2/3}}$$

**Maximize**:

$$M = \left[\frac{\rho}{\left(\phi_B \sigma_f\right)^{2/3}}\right]$$
Material indices with shape factors change material selection

<table>
<thead>
<tr>
<th>Material</th>
<th>$\sigma_f$ (MPa)</th>
<th>$\rho$ (Mg/m³)</th>
<th>$\phi_B$</th>
<th>$\sigma_f^{2/3}/\rho$</th>
<th>$(\phi_B \sigma_f)^{2/3}/\rho$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spruce (Norwegian)</td>
<td>80</td>
<td>0.51</td>
<td>1</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>Bamboo</td>
<td>120</td>
<td>0.7</td>
<td>2.2</td>
<td>35</td>
<td>59</td>
</tr>
<tr>
<td>Steel (Reynolds 531)</td>
<td>880</td>
<td>7.82</td>
<td>7.5</td>
<td>12</td>
<td>45</td>
</tr>
<tr>
<td>Alu (6061-T6)</td>
<td>250</td>
<td>2.7</td>
<td>5.9</td>
<td>15</td>
<td>48</td>
</tr>
<tr>
<td>Titanium 6-4</td>
<td>955</td>
<td>4.42</td>
<td>5.9</td>
<td>22</td>
<td>72</td>
</tr>
<tr>
<td>Magnesium AZ 61</td>
<td>165</td>
<td>1.8</td>
<td>4.25</td>
<td>17</td>
<td>44</td>
</tr>
<tr>
<td>CFRP</td>
<td>375</td>
<td>1.5</td>
<td>4.25</td>
<td>35</td>
<td>91</td>
</tr>
</tbody>
</table>

*Material Index w/out shape factor

**Material Index with shape factor
Strength Constraint

Stiffness Constraint

### Example of Material Selection including Shape: Floor Joists

<table>
<thead>
<tr>
<th>Material for floor joists</th>
<th>Wood beam</th>
<th>Steel I-beam</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Density (g/cm³)</strong></td>
<td>~0.58</td>
<td>~7.9</td>
</tr>
<tr>
<td><strong>Modulus (GPa)</strong></td>
<td>~10</td>
<td>~210</td>
</tr>
<tr>
<td><strong>Material Cost ($/kg)</strong></td>
<td>~$0.90</td>
<td>~$0.65</td>
</tr>
<tr>
<td><strong>φ&lt;sub&gt;E&lt;/sub&gt;</strong></td>
<td>2.0-2.2</td>
<td>15-25</td>
</tr>
<tr>
<td><strong>E&lt;sup&gt;1/2&lt;/sup&gt;/C&lt;sub&gt;m&lt;/sub&gt;ρ</strong></td>
<td>~6.1</td>
<td>~2.8</td>
</tr>
<tr>
<td><strong>(φ&lt;sub&gt;E&lt;/sub&gt;Er/2)</strong></td>
<td>~8.8</td>
<td>~12.6</td>
</tr>
</tbody>
</table>

*Material Index w/ out shape factor

**Material Index with shape factor