### TABLE II. Characteristics of Cubic Lattices

<table>
<thead>
<tr>
<th></th>
<th>Simple</th>
<th>Body-Centered</th>
<th>Face-Centered</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Cell Volume</td>
<td>$a^3$</td>
<td>$a^3$</td>
<td>$a^3$</td>
</tr>
<tr>
<td>Lattice Points Per Cell</td>
<td>1</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>Nearest Neighbor Distance</td>
<td>$a$</td>
<td>$\frac{a\sqrt{3}}{2}$</td>
<td>$\frac{a}{\sqrt{2}}$</td>
</tr>
<tr>
<td>Number of Nearest Neighbors</td>
<td>6</td>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>Second Nearest Neighbor Distance</td>
<td>$a\sqrt{2}$</td>
<td>$a$</td>
<td>$a$</td>
</tr>
<tr>
<td>Number of Second Neighbors</td>
<td>12</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>$a = f(r)$</td>
<td>$2r$</td>
<td>$4r/\sqrt{3}$</td>
<td>$2\sqrt{2}r$</td>
</tr>
<tr>
<td>or $4r =$</td>
<td>$\sqrt{4}a$</td>
<td>$\sqrt{3}a$</td>
<td>$\sqrt{2}a$</td>
</tr>
<tr>
<td>packing density</td>
<td>0.52</td>
<td>0.68</td>
<td>0.74</td>
</tr>
</tbody>
</table>
Crystallographic Notation

**position**: \(x, y, z\), coordinates, separated by commas, no enclosure.

- \(\textbf{O}: 0,0,0\)
- \(\textbf{A}: 0,1,1\)
- \(\textbf{B}: 1,0,\frac{1}{2}\)

**direction**: move coordinate axes so that line passes through origin.
- Define vector from \(\textbf{O}\) to point on the line.
- Choose smallest set of integers.
- No commas, enclose in brackets, clear fractions.

\[
\begin{align*}
\text{OB} & \rightarrow 1 \ 0 \ \frac{1}{2} \text{ clear fractions} \ [201] \\
\text{AO} & \rightarrow [0\overline{1}\overline{1}] \text{ minus denoted by macaron}
\end{align*}
\]

Can denote entire family of directions by carats \(<>\).

- E.g., all body diagonals: \(<111> = [111], [\overline{1}11], [\overline{1}\overline{1}1], [1\overline{1}1]\), etc.
- All cube edges: \(<001>\)
- All face diagonals: \(<011>\)
- All body diagonals: \(<111>\)
**plane**: Miller\(^1\) indices – recall equation of a plane in space

\[
\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1, \text{ where } a, b, c \text{ are intercepts of the plane with the } \\
x, y, z \text{ axes, respectively}
\]

- let \( h = \frac{1}{a}, \ k = \frac{1}{b}, \) and \( l = \frac{1}{c}, \) so that \( hx + ky + lz = 1 \)

- no commas\(^2\), enclose in parentheses \((hkl)\)

- can denote entire family of planes by braces \(\{\}\)  
  e.g., all faces of unit cell: \(\{001\} = (001), (00\bar{1}), (\bar{1}00), (0\bar{1}0), \) etc.

- cool property: \( (hkl) \perp [hkl] \)

\(^1\) William Hallowes Miller, British mineralogist, 1839  
\(^2\) plane must not include the origin
Intercept at $\infty$

Intercept at $b$

Intercept at $\frac{1}{2}a$

$\left(\frac{1}{2},1,\infty\right)$

Image by MIT OpenCourseWare.
Miller indices \((hkl)\):
\[
\begin{bmatrix}
1 \\
1/2 \\
1 \\
\infty
\end{bmatrix}
\]
(210)
Intercept at $\infty$

Miller indices (\(hkl\)):

\[
\begin{bmatrix}
\frac{1}{2} \\
1 \\
1
\end{bmatrix} = (210)
\]
Move the origin out of the plane
Image by MIT OpenCourseWare.
\( a = b = c = "a" \)

\[
d_{020} = \frac{a}{(0^2 + 2^2 + 0^2)^{1/2}} = \frac{a}{2}
\]
\[ d_{111} = \frac{a}{\left(1^2 + 1^2 + 1^2\right)^{1/2}} = \frac{a}{\sqrt{3}} \]
## Ionization Energies (eV)

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>14</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>He</td>
<td>25</td>
<td>55</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Li</td>
<td>5</td>
<td>76</td>
<td>123</td>
<td>9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Be</td>
<td>9</td>
<td>18</td>
<td>154</td>
<td>218</td>
<td>16</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>8</td>
<td>25</td>
<td>38</td>
<td>260</td>
<td>341</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>11</td>
<td>24</td>
<td>48</td>
<td>64</td>
<td>393</td>
<td>491</td>
<td>36</td>
</tr>
<tr>
<td>N</td>
<td>14</td>
<td>30</td>
<td>48</td>
<td>78</td>
<td>98</td>
<td>523</td>
<td>668</td>
</tr>
</tbody>
</table>

\[ E_1 = -KZ^2 \]
First Nobel Prize in Physics (1901)