Session #24: Homework Solutions

Problem #1

To increase its corrosion resistance, chromium (Cr) is diffused into steel at 980°C. If during diffusion the surface concentration of chromium remains constant at 100%, how long will it take (in days) to achieve a Cr concentration of 1.8% at a depth of 0.002 cm below the steel surface? \( D_0 = 0.54 \text{ cm}^2/\text{s}; \ E_A = 286 \text{ kJ/mol} \)

Solution

A solution to Fick’s second law for the given boundary conditions is:

\[
\frac{C}{C_s} = 1 - \text{erf} \frac{x}{2\sqrt{D}t}, \quad \text{from which we get} \quad \text{erf} \frac{x}{2\sqrt{D}t} = 1 - 0.018 = 0.982
\]

From the error function tables, 0.982 is the erf of 1.67. This means that

\[
\frac{0.002}{2\sqrt{D}t} = 0.001 = 1.67
\]

\[
D = D_0 e^{\left(-\frac{286 \times 10^5}{8.314 \times 1253}\right)} = 6.45 \times 10^{-13} \text{ cm}^2/\text{s}
\]

\[
\therefore \ t = \frac{0.001^2}{1.67^2 \times 6.45 \times 10^{-13}} = 5.56 \times 10^5 \text{ sec} = 6.4 \text{ days}
\]

Problem #2

By planar diffusion of antimony (Sb) into p-type germanium (Ge), a p-n junction is obtained at a depth of \( 3 \times 10^{-3} \) cm below the surface. What is the donor concentration in the bulk germanium if diffusion is carried out for three hours at 790°C? The surface concentration of antimony is held constant at a value of \( 8 \times 10^{18} \) cm\(^{-3}\); \( D_{790°C} = 4.8 \times 10^{-11} \text{ cm}^2/\text{s} \).

Solution
\[
\frac{c}{c_s} = \frac{\text{erfc}}{2\sqrt{Dt}} = \text{erfc} \frac{3 \times 10^{-3}}{2\sqrt{Dt}} = \text{erfc}(2.083)
\]

\[
\frac{C}{C_s} = 1 - \text{erf}(2.083), \quad 1 - \frac{C}{C_s} = 0.9964
\]

\[
\frac{C}{C_s} = 3.6 \times 10^{-3}, \quad c = 2.88 \times 10^{16} \text{ cm}^{-3}
\]

The donor concentration in germanium is \(2.88 \times 10^{16}/\text{cm}^3\).

**Problem #3**

You wish to dope a single crystal of silicon (Si) with boron (B). The specification reads \(5 \times 10^{16}\) boron atoms/cm\(^3\) at a depth of 25 \(\mu\)m from the surface of the silicon. What must be the effective concentration of boron in units of atoms/cm\(^3\) if you are to meet this specification within a time of 90 minutes? Assume that initially the concentration of boron in the silicon crystal is zero. The diffusion coefficient of boron in silicon has a value of \(7.23 \times 10^{-9}\) cm\(^2\)/s at the processing temperature.

**Solution**

\[c(x, t) = A + B \text{erf} \frac{x}{2\sqrt{Dt}} \quad ; \quad c(0, t) = c_s = A \quad ; \quad c(x, 0) = c_i = 0\]

\[c(\infty, t) = c_i = 0 = A + B \rightarrow A = -B\]

\[
\therefore c(x, t) = c_s - c_s \text{erf} \frac{x}{2\sqrt{Dt}} = c_s \text{erfc} \frac{x}{2\sqrt{Dt}} \rightarrow 5 \times 10^{16} = c_s \text{erfc} \frac{25 \times 10^{-4}}{2\sqrt{7.23 \times 10^{-9} \times 90 \times 60}}
\]

\[
\therefore c_s = \frac{5 \times 10^{16}}{25 \times 10^{-4}} = 6.43 \times 10^{16} \text{ cm}^{-3}
\]

\[
\text{erfc}(0.20) = 1 - \text{erf}(0.20) = 1 - 0.2227 = 0.7773
\]

**Problem #4**

A slab of plate glass containing dissolved helium (He) is placed in a vacuum furnace at a temperature of 400°C to remove the helium from the glass. Before vacuum treatment, the concentration of helium is constant throughout the glass. After 10 minutes in vacuum at 400°C, at what depth from the surface of the glass has the concentration of helium decreased to 1/3 of its initial value? The diffusion coefficient of helium in the plate glass at the processing temperature has a value of \(3.091 \times 10^{-6}\) cm\(^2\)/s.
Solution

\[ c = A + B \text{erf} \frac{x}{2\sqrt{Dt}}; \quad c(0, t) = 0 = A; \quad c(\infty, t) = c_o = B \]

\[ \therefore c(x, t) = c_0 \text{erf} \frac{x}{2\sqrt{Dt}} \]

What is \( x \) when \( c = c_o/3 \)?

\[ \frac{c_o}{3} = \frac{c_o \text{erf} \frac{x}{2\sqrt{Dt}}}{2\sqrt{Dt}} \rightarrow 0.33 = \text{erf} \frac{x}{2\sqrt{Dt}} \quad \text{erf}(0.30) = 0.3286 \approx 0.33 \]

\[ \therefore \frac{x}{2\sqrt{Dt}} = 0.30 \rightarrow x = 2 \times 0.30 \times \frac{\sqrt{3.091 \times 10^{-6} \times 10 \times 60}}{} = 2.58 \times 10^{-2} \text{ cm} = 258 \mu\text{m} \]