Session #2: Homework Solutions

Problem #1

In all likelihood, the Soviet Union and the United States together in the past exploded about ten hydrogen devices underground per year.

(a) If each explosion converted about 10 g of matter into an equivalent amount of energy (a conservative estimate), how many kJ of energy were released per device?

(b) If the energy of these ten devices had been used for propulsion to substitute for gasoline combustion, how many gallons of gasoline would not have had to be burned per year? (One gallon of gasoline releases about $1.5 \times 10^5$ kJ during combustion.)

Solution

Required:  $\Delta E = mc^2$

(a) $\Delta E = mc^2 = 10 \text{ g} \times \frac{1 \text{ kg}}{1000 \text{ g}} \times (3 \times 10^8 \text{ m/s})^2$

\[ = 9 \times 10^{14} \text{ kg} \text{ m}^2\text{s}^{-2} = 9 \times 10^{14} \text{ J} = 9 \times 10^{11} \text{ kJ/bomb} \]

(b) $E_{\text{total}} = 10 \times 9 \times 10^{11} = 9 \times 10^{12} \text{ kJ/year}$

No. gallons of gasoline saved = \[ \frac{1 \text{ gal}}{1.5 \times 10^5 \text{ kJ}} \times 9 \times 10^{12} \text{ kJ/year} \]

\[ = 6 \times 10^7 \text{ gallons/year} \]

Problem #2

How much oxygen (in kg) is required to completely convert 1 mole of C$_2$H$_6$ into CO$_2$ and H$_2$O?

Solution

To get the requested answer, let us formulate a "stoichiometric" equation (molar quantities) for the reaction: $\text{C}_2\text{H}_6 + 70 \rightarrow 2\text{CO}_2 + 3\text{H}_2\text{O}$. Each C$_2$H$_6$ (ethane) molecule requires 7 oxygen atoms for complete combustion. In molar quantities: 1 mole of C$_2$H$_6$ = 2 x 12.01 + 6 x 1.008 = 30.07 g requires

$7 \times 15.9984 \text{ g} = 1.12 \times 10^2 \text{ oxygen} = 0.112 \text{ kg oxygen}$

We recognize the oxygen forms molecules, O$_2$, and therefore a more appropriate formulation would be: $\text{C}_2\text{H}_6 + 7/2 \text{O}_2 \rightarrow 2\text{CO}_2 + 3\text{H}_2\text{O}$. The result would be the same.
Problem # 3

A nucleus of mass number 56 contains 30 neutrons. An “ion” of this element has 23 electrons. Write the symbol of this ion and give the ionic charge as a superscript on the right.

Solution

\[ \frac{56}{26}A^{+++} = \frac{56}{26}Fe^{+++} \]

Problem # 4

Magnesium (Mg) has the following isotopic distribution:

- \[ ^{24}_{24}Mg \] 23.985 amu at 0.7870 fractional abundance
- \[ ^{25}_{25}Mg \] 24.986 amu at 0.1013 fractional abundance
- \[ ^{26}_{26}Mg \] 25.983 amu at 0.1117 fractional abundance

What is the atomic weight of magnesium (Mg) according to these data?

Solution

The atomic weight is the arithmetic average of the atomic weights of the isotopes, taking into account the fractional abundance of each isotope.

\[
\text{At. Wt.} = \frac{23.985 \times 0.7870 + 24.986 \times 0.1013 + 25.983 \times 0.1117}{0.7870 + 0.1013 + 0.1117} = 24.310
\]

Problem # 5

(a) Balance the equation for the reaction between CO and \( O_2 \) to form \( CO_2 \).

(b) If 32.0 g of oxygen react with CO to form carbon dioxide (\( CO_2 \)), how much CO was consumed in this reaction?

Solution

(a) \[ CO + \frac{1}{2} O_2 \rightarrow CO_2 \]

(b) [Information only at 1 digit!]

Molecular Weight (M.W.) of \( O_2 \): 32.0
Molecular Weight (M.W.) of \( CO \): 28.0

Available oxygen: 32.0 g = 1 mole, correspondingly the reaction involves 2 moles of \( CO \) [see (a)]:

\[ O_2 + 2 \text{ CO} \rightarrow 2 \text{ CO}_2 \]

Mass of CO reacted = 2 moles \times 28 \text{ g/mole} = \textbf{56.0 g}
Problem # 6

One mole of electromagnetic radiation (light, consisting of energy packages called photons) has an energy of 171 kJ/mole photons.
(a) Determine the wavelength of this light and its position in the visible spectrum.

(b) Determine the frequency of this radiation (in SI units).

Solution

(We know: \( E_{\text{photon}} = h\nu = \frac{hc}{\lambda} \) to determine the wavelength associated with a photon we need to know its energy).

(a) \[
E = \frac{171\text{kJ}}{\text{mole}} = \frac{1.71 \times 10^5 \text{J}}{\text{mole}} \times \frac{1\text{ mole}}{6.02 \times 10^{23}\text{ photons}} = \frac{2.84 \times 10^{-19}\text{J}}{\text{photon}}; \quad E_{\text{photon}} = 2.84 \times 10^{-19}\text{ J} = h\nu = \frac{hc}{\lambda}
\]

\[
\lambda = \frac{hc}{E_{\text{photon}}} = \frac{6.63 \times 10^{-34}\text{Js}}{2.84 \times 10^{-19}\text{J}} = 7.00 \times 10^{-7}\text{m} = 700\text{ nm (red light)}
\]

(b) (IS or SI units are in m, k, s)

\[
\nu = c \frac{\lambda}{c} = \frac{3 \times 10^8 \text{m/s}}{7.00 \times 10^{-7}\text{ m}} = 4.29 \times 10^{14}\text{s}^{-1} = 4.29 \times 10^{14}\text{ Hz}
\]

Problem # 7

Determine the velocity of an electron (in m/s) that has been subjected to an accelerating potential \( V \) of 150 Volt. (The energy imparted to an electron by an accelerating potential of one Volt is \( 1.6 \times 10^{-19} \) Joules; dimensional analysis shows that the dimensions of charge x potential correspond to those of energy; thus: 1 electron Volt (1eV) = \( 1.6 \times 10^{-19} \) Coulomb x 1 Volt = \( 1.6 \times 10^{-19} \) Joules.)

Solution

We know: \( E_{\text{kin}} = \frac{mv^2}{2} = e \times V \) (charge applied potential)

\( m_e = 9.1 \times 10^{-31} \) kg
\[ E_{\text{kin}} = e \times V = \frac{mv^2}{2} \]

\[ v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 150}{9.1 \times 10^{-31}}} = 7.26 \times 10^6 \text{ m/s} \]

**Problem # 8**

Determine in units of eV the energy of a photon (hν) with the wavelength of 800 nm.

**Solution**

\[
E_{(eV)} = \frac{hc}{\lambda} \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} = \frac{6.63 \times 10^{-34} \text{ [Js]} \times 3 \times 10^8 \text{[m/s]}}{8.00 \times 10^{-7} \text{ m}} \times \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}}
\]

\[ = 1.55 \]