Statistics in Materials Testing

- Basic statistical measures

  arithmetic mean \[ \bar{x}_f = \frac{1}{N} \sum_{i=1}^{N} \sigma_{f,i} \]

  standard deviation \[ s = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (\bar{x}_f - \sigma_{x,i})^2} \]

Room-temperature tensile strength of a graphite/epoxy composite (P. Shyprykevich, ASTM STP 1003, pp. 111–135, 1989.) (in kpsi): 72.5, 73.8, 68.1, 77.9, 65.5, 73.23, 71.17, 79.92, 65.67, 74.28, 67.95, 82.84, 79.83, 80.52, 70.65, 72.85, 77.81, 72.29, 75.78, 67.03, 72.85, 77.81, 75.33, 71.75, 72.28, 79.08, 71.04, 67.84, 69.2, 71.53.

\[ \bar{x}_f = 73.28, \quad s = 4.63 \text{ (kpsi)} \]

The coefficient of variation is C.V. = \( \frac{4.63}{73.28} \times 100\% = 6.32\% \).

- The normal distribution

![Histogram and normal distribution functions.](image)

Figure 1: Histogram and normal distribution functions.
\[ f(X) = \frac{1}{\sqrt{2\pi}} \exp \left( -\frac{X^2}{2} \right), \quad X = \frac{\sigma_f - \overline{f}}{s} \]

Cumulative probability

<table>
<thead>
<tr>
<th>±x/s</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>68.3</td>
</tr>
<tr>
<td>1.96</td>
<td>95.0</td>
</tr>
<tr>
<td>2</td>
<td>95.8</td>
</tr>
<tr>
<td>3</td>
<td>99.7</td>
</tr>
</tbody>
</table>

Figure 2: Cumulative probability plot.

- Confidence limits

\[ s_m = \frac{s}{\sqrt{N}} \]

Since 95\% of all measurements of a normally distributed population lie within 1.96 standard deviations from the mean, the ratio ±1.96s/√N is the range over which we can expect 95 out of 100 measurements of the mean to fall.
Goodness of fit

\[
\chi^2 = \sum \frac{(\text{expected} - \text{observed})^2}{\text{observed}}
= \sum_{i=1}^{N} \frac{(Np_i - n_i)^2}{n_i}
\]

where \(N\) is the total number of specimens, \(n_i\) is the number of specimens actually failing in a strength increment \(\Delta \sigma_{f,i}\) and \(p_i\) is the probability expected from the assumed distribution of a specimen having having a strength in that increment.

<table>
<thead>
<tr>
<th>Lower Limit</th>
<th>Upper Limit</th>
<th>Observed Frequency</th>
<th>Expected Frequency</th>
<th>Chisquare</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>69.33</td>
<td>7</td>
<td>5.9</td>
<td>0.198</td>
</tr>
<tr>
<td>69.33</td>
<td>72.00</td>
<td>5</td>
<td>5.8</td>
<td>0.116</td>
</tr>
<tr>
<td>72.00</td>
<td>74.67</td>
<td>8</td>
<td>6.8</td>
<td>0.214</td>
</tr>
<tr>
<td>74.67</td>
<td>77.33</td>
<td>2</td>
<td>5.7</td>
<td>2.441</td>
</tr>
<tr>
<td>77.33</td>
<td>\infty</td>
<td>8</td>
<td>5.7</td>
<td>0.909</td>
</tr>
</tbody>
</table>

\(\chi^2 = 3.878\)

The number of degrees of freedom for this Chi-square test is 4; this is the number of increments less one, since we have the constraint that \(n_1 + n_2 + n_3 + n_5 = 30\). From Table 3 in Appendix H, we read that \(\alpha = 0.05\) for \(\chi^2 = 9.488\), where \(\alpha\) is the fraction of the \(\chi^2\) population with values of \(\chi^2\) greater than 9.488.

The “B-allowable.”

The “B-allowable” strength is the stress level for which we have 95% confidence that 90% of all specimens will have at least that strength.

\[
B = \overline{\sigma_f} - k_B \sigma
\]

where \(k_b\) is \(n^{-1/2}\) times the 95th quantile of the “noncentral t-distribution;” this factor is tabulated, but can be approximated by the formula
\[ k_b = 1.282 + \exp(0.958 - 0.520 \ln N + 3.19/N) \]

In the case of the previous 30-test example, \( k_B \) is computed to be 1.78, so this is less conservative than the 3s guide. The B-basis value is then

\[ B = 73.28 - (1.78)(4.632) = 65.05 \]

• The Weibull distribution

![Weibull plot of strength data.](image)

Figure 3: Weibull plot of strength data.

\[ \ln P_s = -\left( \frac{\sigma}{\sigma_0} \right)^m \]

\[ \ln(\ln P_s) = -m \ln \left( \frac{\sigma}{\sigma_0} \right) \]
Hence the double logarithm of the probability of exceeding a particular strength $\sigma$ versus the logarithm of the strength should plot as a straight line with slope $m$.

The Weibull equation can be used to predict the magnitude of the size effect. If for instance we take a reference volume $V_0$ and express the volume of an arbitrary specimen as $V = n V_0$, then the probability of failure at volume $V$ is found by multiplying $P_s(V)$ by itself $n$ times:

$$P_s(V) = [P_s(V_0)]^n = [P_s(V_0)]^{V/V_0}$$

$$P_s(V) = \exp \left( -\frac{V}{V_0} \left( \frac{\sigma}{\sigma_0} \right)^m \right)$$

Hence the probability of failure increases exponentially with the specimen volume.

- Remember Mark Twain’s aphorism:

  *There are lies, damned lies, and statistics.*
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