3.15 - Problem Set 2 Solutions

Problem 1

a. Light produces excess carriers at surface.

intrinsic so \( n = p \).
Area under curve decreases due to recombination. Curve spreads due to diffusion.

\[
\frac{\partial n}{\partial t} = \frac{\partial n}{\partial t_{\text{drift}}} + \text{diffusion} + \text{R&G}
\]

\[
\frac{\partial n}{\partial t_{\text{drift}}} = 0: \text{No fields}
\]

Diffusion = \( \frac{1}{e} \nabla J = D_n \frac{\partial^2 n}{\partial x^2} \)

R & G: \( \frac{dn}{dt} = -\frac{n_e}{\tau} \) where \( n_e \) is the excess carrier concentration.

\[
\frac{dn}{dt} = D_n \frac{\partial^2 n}{\partial x^2} + \left( \frac{n_e}{\tau} \right)
\]

Same type of expression for holes.

\( \tau \) is due to recombination at traps, since it’s intrinsic material. These are more effective than band to band recombination, but there are relatively few of them. \( \tau = 1/r_2N_T \). \( r_2 \gg \tau \) but \( N_T \) less than typical doping levels.
Problem 2

a.

Light is absorbed through semiconductor. Recombination and generation occur at all places inside the semiconductor but generation is strongest at the surface.

More carriers created near surface
Diffusion away from surface
Also, no drift since no $E$ ($n = p$ everywhere), neglecting difference in mobilities.

b. $5 \times 10^{-4} W/cm^2$. Each photon is $\lambda = 0.75 \mu m \Rightarrow 1.65 \text{ eV}$ ($1 \text{ eV} = 1.24 \mu m$).

$\Rightarrow$ # photons$/cm^2$s.

$$\frac{5 \times 10^{-4}}{1.65 \cdot 1.6 \cdot 10^{-19}} = 1.9 \cdot 10^{15}/cm^2 \cdot s$$

c.

$$\frac{I}{I_0} = 0.05 = \exp(-\alpha \cdot t)$$
\[ \alpha \cdot t = -\ln(0.05) \]
\[ t = \left( \frac{\ln 20}{500} \right) cm \approx 0.1 mm \]

Most absorption occurs near the surface.

d.

Since \( E_{\text{photon}} > E_g \), each photon makes 1 electron-hole pair. \( 2 \times 10^{15} \) e-h pairs/cm\(^2\)s \( \times 6\text{cm}^2 = 1.1 \times 10^{16}/s. \)

e.

Excess carriers survive \( 2 \times 10^{-4} \) seconds. In this time \( 2.4 \times 10^{12} \) are produced in a volume of \( 2 \times 3 \times 0.3\text{cm}^3 \). Average \( \delta n = \frac{2.4 \times 10^{12}}{1.8} = 1.3 \times 10^{12}\text{cm}^{-3} \)

The intrinsic \( n_i \) for GaAs is \( \approx 2 \cdot 10^6 \text{cm}^{-3} \).

\[ \sigma = ne(\mu_e + \mu_h) = 1.3 \cdot 10^{12} \cdot 1.6 \cdot 10^{-19} \cdot 8900 \frac{1}{cm^3} \cdot \frac{cm^2}{Vs} \]

GaAs has \( n_i \approx 2 \cdot 10^6 \text{cm}^{-3} \). When light shines, \( n_i = Pl = 10^{12}\text{cm}^{-3} \) so the conductivity has increased by \( \approx 10^6 \).
Problem 3

a. Equilibrium: again, fluxes in depl. region are equal and opposite.

Biasing positive on left leads to a large electron current.

Bias other way: large hole diffusion current. Characteristic shows exponential current increase in both directions.
Problem 4.

a.

I define $p_{n,0}$ to be the expression for equilibrium hole concentration in the $n^+$ material. $p_{n,0} = n_i^2/N_D$. Note that since the material is $n^+$, $N_D$ is assumed to be very high ($> 10^{19}$), so $p_{n,0}$ will be very small.

The peak-height is determined readily from the “law of the junction” for carrier concentration at the junction edges in non-equilibrium conditions. Law of the junction:

$$n_p = n_i^2 \exp(qV_A/kT)$$
[see Pierret p. 245]

So, our peak-height will be:

$$p_n(x = 0) = p_{n,0} \exp(qV_A/kT)$$

$V_A$ is the forward bias voltage being applied to the junction.

Minority carrier concentration will fall exponentially from $p_n(x = 0)$ to $p_{n,0}$ along the length from the depletion region edge into $n^+$ bulk. $L_p$ is the diffusion length of holes that characterizes that fall, defined as $L_p = \sqrt{\tau_p D_p}$, where $\tau_p$ is minority carrier lifetime bulk and $D_p$ is the diffusion coefficient. From Pierret page 116, we know that a typical value for $\tau_p$ is 0.5 microseconds. The $n^+$ region is heavily doped, so if we assume $N_D$ to be equal to $10^{19}$, mobility (according to the plot on Pierret p. 80) would be about 70$[cm^2/V - s]$. $D_p = kT/q \times u = 0.026 \times 70 = 1.82[cm^2/sec]$. We can then estimate $L_p = \sqrt{5 \cdot 10^{-6} \times 1.82cm} = 9.5\mu m$.

b.

In the ideal diode, we assume there is no voltage drop and resultantly no electric field in the bulk. Carrier movement in this region is entirely driven by diffusion,
as carriers flow down the concentration gradient.

c. 

$L_p$ will decrease by a factor of $\sqrt{10}$, causing this exponential decrease of carrier concentration in the material to occur over a shorter distance. See above plot. 

$L'_p = \frac{L_p}{\sqrt{10}}$.

d. 

The boundary condition at the end of the n-side is that of an ohmic contact, which forces carrier concentrations to their equilibrium value. This will result in an approximately linear carrier concentration profile:

e. 

The boundary condition at the ohmic contact forces a larger carrier-concentration gradient in the short diode, which will result in a larger current.
Problem 5.

\[ N_A = N_D \text{ so } d_p = d_A \]

\[ e = \frac{N_A}{d_p} \text{ ex from } x = -d_A \text{ to } x = d_p \]

\[ e = \frac{1}{\varepsilon_D \varepsilon_r} \int p(x) dx = \frac{1}{\varepsilon_D \varepsilon_r} \int_{-d_n}^{d_n} \frac{N_A}{d_n} x dx \text{ up to } x = d_p \]

\[ e = \frac{-e N_A}{2 \varepsilon_0 \varepsilon_r d_p} (x^2 - d_n^2) \text{ Parabolic} \]
Potential:

\[ V = -\int_\infty^x \epsilon(x) dx \]
\[ = -\int_{-d_A}^x \frac{eN_A}{2\epsilon_0\epsilon_r d_p} (x^2 - d_n^2) dx \]
\[ = \frac{eN_A}{2\epsilon_0\epsilon_r d_p} \left( \frac{x^3}{3} - d_n^2 x - \frac{2d_n^3}{3} \right) \]

This is cubic.