Exam 2 (5 pages)
Closed book exam. Formulae and data are on the last 3.5 pages of the exam. This takes **80 min** and there are 80 points total. Be brief in your answers and use sketches.

**Assume everything is at 300K unless otherwise noted.**

1. **MOSFET [20 points]**
   A MOSFET has the following structure:
   
   ![MOSFET Diagram]

   a) What happens when you apply a voltage $V_G$ to G (when S and D are grounded)? Consider both positive and negative voltages. Illustrate with a sketch of the MOS band diagram. (10)

   b) What happens when you apply a negative voltage $V_D$ to D, for different values of $V_G$ (zero, positive and negative)? (assume S is grounded.) Draw plots of current $I_{SD}$ vs $V_D$ for different values of $V_G$. (10)

2. **Optics [35 points]**
   a) Draw a diagram of the attenuation of a silica optical fiber vs. wavelength, and explain the shape of the curve. (7)

   b) Describe three sources of dispersion in a fiber (one sentence each). (6)

   c) We need to design a system to deliver high power laser light of energy 2 eV via a fiber for surgery inside the body. Would you be concerned with dispersion and loss in this application? (4)

   d) Select materials for the core, cladding and substrate of the 2 eV laser, explaining your choices. If there is more than one option, which would be preferable? (8)

   e) It would be nice to have a laser based on Si or Si$_x$Ge$_{1-x}$ ($0 \leq x \leq 1$) because this would be compatible with other silicon devices. What colors of light could you expect from a laser made from SiGe? What is the difficulty with making such a laser? How could this be overcome, and what quality output would the laser produce? (Be concise in this question – no more than 5-6 sentences.) (10)
3. Heterostructures [25 points]
a) Explain concisely the conditions under which a system can act as laser. (No more than 4-5 sentences). Illustrate by describing a ruby or a Nd-YAG laser. (12)

b) Why is a heterostructure better than a homostructure for making a semiconductor laser? (7)

c) A band diagram of a heterostructure is given below. What can you deduce from this diagram about the doping levels of materials A and B? What has happened to materials A and B near the interface? (6)

![Band diagram of a heterostructure](image)

**Data and Formulae**

![Energy Gap and Lattice Constants](image)

Figure by MIT OCW.
<table>
<thead>
<tr>
<th>Properties</th>
<th>Si</th>
<th>GaAs</th>
<th>SiO2</th>
<th>Ge</th>
</tr>
</thead>
<tbody>
<tr>
<td>Atoms/cm³, molecules/cm³ x 10²²</td>
<td>5.0</td>
<td>4.42</td>
<td>2.27a</td>
<td>amorphous</td>
</tr>
<tr>
<td>Structure</td>
<td>diamond</td>
<td>zincblende</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lattice constant (nm)</td>
<td>0.543</td>
<td>0.565</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Density (g/cm³)</td>
<td>2.33</td>
<td>5.32</td>
<td>2.27a</td>
<td></td>
</tr>
<tr>
<td>Relative dielectric constant, εᵣ</td>
<td>11.9</td>
<td>13.1</td>
<td>3.9</td>
<td></td>
</tr>
<tr>
<td>Permittivity, ε = εᵣε₀</td>
<td>1.05</td>
<td>1.16</td>
<td>0.34</td>
<td></td>
</tr>
<tr>
<td>Expansion coefficient (dL/LdT) x 10⁻¹²</td>
<td>2.6</td>
<td>6.86</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Specific Heat (joule/g K)</td>
<td>0.7</td>
<td>0.35</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Thermal conductivity (watt/cm K)</td>
<td>1.48</td>
<td>0.46</td>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td>Thermal diffusivity (cm²/sec)</td>
<td>0.9</td>
<td>0.44</td>
<td>0.006</td>
<td></td>
</tr>
<tr>
<td>Energy Gap (eV)</td>
<td>1.12</td>
<td>1.424</td>
<td>-9</td>
<td>0.67</td>
</tr>
<tr>
<td>Drift mobility (cm²/volt-sec)</td>
<td>1500</td>
<td>8500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effective density of states</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(cm⁻³) x 10¹⁹</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conduction band</td>
<td>2.8</td>
<td>0.047</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Valence band</td>
<td>1.04</td>
<td>0.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intrinsic carrier concentration (cm⁻³)</td>
<td>1.45 x 10¹⁰</td>
<td>1.79 x 10⁶</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Useful equations

\[ g_0(E) = \frac{m^*}{(2\pi \hbar^2) \frac{d^2}{(E-E_0)^2}} \]
\[ g_v(E) = \frac{m^*}{(2\pi \hbar^2) \frac{d^2}{(E_v-E)^2}} \]
\[ f(E) = \frac{1}{N_c \exp \left( E - E_f \right) / kT} + \frac{1}{N_v \exp \left( E_f - E_v \right) / kT} \]
\[ n = n_e \exp \left( E_f - E_0 \right) / kT, \quad p = n_h \exp \left( E_f - E_v \right) / kT \]
\[ n_i = N_c \exp \left( E_f - E_0 \right) / kT \quad \text{where} \ N_c = 2 \left( 2 \pi m^* kT / h^2 \right)^{3/2} \]
\[ n_i = n_e^2 \text{ at equilibrium} \]
\[ n_e^2 = N_c N_v \exp \left( E_f - E_0 \right) / kT = N_c N_v \exp \left( -E_v \right) / kT \]
\[ E_f - E_0 = kT \ln \left( n_e / n_i \right) = -kT \ln \left( p_i / n_i \right) \]
\[ \sim kT \ln \left( N_c / N_v \right) \text{ ntype or } -kT \ln \left( N_v / N_c \right) \text{ ptype} \]

Drift thermal velocity \( v_d = \mu E \) \( E \) = field

Current density (electrons) \( J = n e v_d \)

Current density (electrons & holes) \( J = e (n \mu_n + p \mu_p) E \)

Conductivity \( \sigma = J / E = \left( n \mu_n + p \mu_p \right) \)

Diffusion \( J = e D_n \nabla n + e D_p \nabla p \)

Einstein relation: \( D_n / \mu_n = kT / e \)

R and G
\[ R = G = n e p = n_i r_0^2 \quad \text{at equilibrium} \]

Fick’s law
\[ \frac{dn}{dt} = -n \frac{d^2}{dr^2} + \frac{dn}{dr} \nabla n + \frac{dn}{dt} \nabla n + \frac{dn}{d\theta} \nabla n \]

so
\[ \frac{dn}{dt} = \left( 1/e \right) \nabla \left( J_{\text{drift}} + J_{\text{diff}} \right) + G - R \]

\[ \frac{dp}{dt} = \left( -p/t_a \right) \text{ or } \frac{dn}{dt} = \left( -p/t_a \right) \text{ or } \frac{dp}{dt} = \left( -p/t_p \right) \]

\[ \tau_a = 1/r_n, \quad \tau_p = 1/r_p \]

If traps dominate \( \tau = 1/r_d N_t \) where \( r_d \gg r \)
\[ E = 1/\varepsilon, \int \rho(x) \, dx \quad \text{where} \quad \rho = e(p - n + N_D - N_A) \]
\[ E = -dV/dx \]
\[ eV_o = (E_t - E_i)_{n\text{-type}} - (E_t - E_i)_{p\text{-type}} \]
\[ = kT/e \ln (n_p/n_n) \text{ or } kT/e \ln (N_A N_D/n_i^2) \]
\[ E = N_{\text{A}} e d_p/\varepsilon_o \varepsilon_r = N_{\text{p}} e d_p/\varepsilon_o \varepsilon_r \quad \text{at } x = 0 \]
\[ V_o = (e/2\varepsilon_o \varepsilon_r) (N_{\text{D}} d_n^2 + N_{\text{A}} d_p^2) \]
\[ d_n = \sqrt{(2e_e \varepsilon_r V_o/e) (N_{\text{D}}/N_{\text{D}} + N_{\text{A}})} \]
\[ d = d_p + d_n = \sqrt{(2e_e \varepsilon_r (V_o + V_{\text{G}})/e) (N_{\text{D}} + N_{\text{A}})/N_{\text{A}} N_{\text{D}}} \]
\[ J = J_o \{ \exp eV/kT - 1 \} \]
\[ I = I_o + I_G \quad V = I \left( R_{\text{PV}} + R_L \right) \]
\[ I = I_o \{ \exp eV/kT - 1 \} + I_G \quad \text{Power} = IV \]

**Photodiode and photovoltaic**

\[ I = I_o + I_G \quad V = I \left( R_{\text{PV}} + R_L \right) \]
\[ I = I_o \{ \exp eV/kT - 1 \} + I_G \quad \text{Power} = IV \]

**Transistor**

- **BJT gain** \( \beta = I_C/I_B \sim I_E/I_B = N_{\text{A,E}}/N_{\text{D,B}} \)
- **JFET**
  \[ V_{\text{SD, sat}} = (e N_{\text{D}} t^2/8\varepsilon_o \varepsilon_r) - (V_o + V_{\text{G}}) \]

**Band structure**

- Effective mass: \( m^* = \hbar^2 (\partial^2 E/\partial k^2)^{-1} \)
- Momentum of an electron typically \( \pi/a \sim 10^{10} \text{ m}^{-1} \)
- Momentum of a photon = \( 2\pi/\lambda \sim 10^7 \text{ m}^{-1} \)
- Uncertainty principle \( \Delta x \Delta p \geq \hbar \)

**Lasers**

- Probability of absorption = \( B_{31} \)
- Stimulated emission = \( A_{31} \)
- Spontaneous emission = \( A_{31} \)

\[ N_3 = N_1 \exp (-h \nu_{31}/kT) \]

Planck's law:
\[ \rho(\nu) d\nu = \{ 8\pi \hbar \nu^3/c^3 \}/\{ \exp (h \nu/kT) - 1 \} \quad \text{dv} \]

\[ B_{13} = B_{31} \]

\[ A_{31}/B_{31} = 8\pi \hbar \nu^3/c^3 \quad \text{(Einstein relations)} \]

**Cavity modes**

\[ \nu = cN/2d, N \text{ an integer} \]

**Fibers**

- Attenuation (dB) = \( \{10/L\} \log(P_{\text{in}}/P_{\text{out}}) \quad L \text{ = fiber length} \)
- Snell's law: \( n \sin \phi = n' \sin \phi' \)
- Dispersion coefficient: \( D_\lambda = -(\lambda/c) (\partial^2 n/\partial \lambda^2)_{\lambda=\lambda_0} \quad \text{ps/km.nm} \)

\[ \sigma_1 = \sigma_0 L D_\lambda \]
# Physical Constants, Conversions, and Useful Combinations

## Physical Constants

- **Avogadro constant**: \( N_A = 6.022 \times 10^{23} \) particles/mole
- **Boltzmann constant**: \( k = 8.617 \times 10^{-5} \text{ eV/K} = 1.38 \times 10^{-23} \text{ J/K} \)
- **Elementary charge**: \( e = 1.602 \times 10^{-19} \text{ coulomb} \)
- **Planck constant**: \( h = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s} = 6.626 \times 10^{-34} \text{ joule} \cdot \text{s} \)
- **Speed of light**: \( c = 2.998 \times 10^{10} \text{ cm/s} \)
- **Permittivity (free space)**: \( \varepsilon_0 = 8.85 \times 10^{-14} \text{ farad/cm} \)
- **Electron mass**: \( m = 9.1095 \times 10^{-31} \text{ kg} \)
- **Coulomb constant**: \( k_c = 8.988 \times 10^9 \text{ newton-m}/(\text{coulomb})^2 \)
- **Atomic mass unit**: \( u = 1.6606 \times 10^{-27} \text{ kg} \)

## Useful Combinations

- **Thermal energy (300 K)**: \( kT = 0.0258 \text{ eV} \approx 1 \text{ eV}/40 \)
- **Photon energy**: \( E = 1.24 \text{ eV at } \lambda = \mu\text{m} \)
- **Coulomb constant**: \( k_c e^2 = 1.44 \text{ eV} \cdot \text{nm} \)
- **Permittivity (Si)**: \( \varepsilon = \varepsilon_r \varepsilon_0 = 1.05 \times 10^{-12} \text{ farad/cm} \)
- **Permittivity (free space)**: \( \varepsilon_0 = 55.3 \varepsilon / \mu\text{m} \)

## Prefixes
- \( \text{k} = \text{kilo} = 10^3 \); \( \text{M} = \text{mega} = 10^6 \); \( \text{G} = \text{giga} = 10^9 \); \( \text{T} = \text{tera} = 10^{12} \)
- \( \text{m} = \text{milli} = 10^{-3} \); \( \text{\mu} = \text{micro} = 10^{-6} \); \( \text{n} = \text{nano} = 10^{-9} \); \( \text{p} = \text{pica} = 10^{-12} \)

## Symbols for Units
- **Ampere (A)**, **Coulomb (C)**, **Farad (F)**, **Gram (g)**, **Joule (J)**, **Kelvin (K)**
- **Meter (m)**, **Newton (N)**, **Ohm (Ω)**, **Second (s)**, **Siemen (S)**, **Tesla (T)**
- **Volt (V)**, **Watt (W)**, **Weber (Wb)**

## Conversions

- \( 1 \text{ nm} = 10^{-9} \text{ m} = 10 \text{ Å} = 10^{-7} \text{ cm} \)
- \( 1 \text{ eV} = 1.602 \times 10^{-9} \text{ Joule} = 1.602 \times 10^{-12} \text{ erg} \)
- \( 1 \text{ eV/particle} = 23.06 \text{ kcal/mol} \)
- \( 1 \text{ newton} = 0.102 \text{ kg-force} \)
- \( 10^6 \text{ newton/m}^2 = 146 \text{ psi} = 10^7 \text{ dyn/cm}^2 \)
- \( 1 \text{ μm} = 10^4 \text{ cm} \)
- \( 0.001 \text{ inch} = 1 \text{ mil} = 25.4 \text{ μm} \)
- \( 1 \text{ bar} = 10^6 \text{ dyn/cm}^2 = 10^5 \text{ N/m}^2 \)
- \( 1 \text{ weber/m}^2 = 10^4 \text{ gauss} = 1 \text{ tesla} \)
- \( 1 \text{ pascal} = 1 \text{ N/m}^2 = 7.5 \times 10^{-3} \text{ torr} \)
- \( 1 \text{ erg} = 10^{-7} \text{ joule} = 1 \text{ dyn-cm} \)

Figure by MIT OCW.