1. The scalar field for the evolution of concentration diffusing from a point source in three dimensions is

\[ c(r, t) = \frac{n_d}{(4\pi D t)^{3/2}} \exp \left( -\frac{r^2}{4Dt} \right) \]  

where \( n_d \) is the number of particles in the point source and \( D \) is the diffusivity.

(a) Find the expression for the vector field \( \vec{J}(r, t) = -D \nabla c \). Use spherical coordinates. \( \vec{J}(r, t) \) is the diffusion flux.

(b) Assume that matter is conserved during diffusion and find an expression for the rate of the accumulation of diffusing particles at any point.

(c) Use words to describe the general trend for how the rate of accumulation varies with time at a fixed point \( r \neq 0 \). Consider times from close to \( t = 0 \) out to a time that is sufficiently long for the concentration field to become essentially negligible at at your fixed point \( r \).

2. Consider a 10 cm rod of Fe–0.8 wt.% C alloy that is initially of uniform composition. At time \( t = 0 \) it is put in contact with a thermal reservoir at 1100 K at one end, and a second thermal reservoir at 1300 K at the other end.

(a) Write out the coupled flux:driving-force equations for heat flux and mass flux in this system. Assume that only the interstitial carbon atoms are mobile at these temperatures.

(b) What conditions must the various coupling coefficients in your equations meet so that the carbon-atom concentration will remain uniform in the bar?

(c) What conditions must the various coupling coefficients in your equations meet so that carbon atoms will be transported toward the hotter end of the bar?