3.225 Electronic and Mechanical Properties of Materials

Test - Elasticity
Friday July 19, 2002

1. A fibre composite is transversely isotropic and has the following elastic constants:

\[ E_1 = E_2 = 5 \text{ GPa} \quad E_3 = 25 \text{ GPa} \quad \nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1} = 0.25 \quad \nu_{13} = -\frac{\varepsilon_3}{\varepsilon_1} = 0.33 \quad G_{13} = 8 \text{ GPa} \]

The composite is subjected to the stress state:

\[ \sigma = \begin{bmatrix} 10 & 0 & 8 \\ 0 & 5 & 3 \\ 8 & 3 & 5 \end{bmatrix} \text{ MPa} \]

Calculate the strain energy in the composite under this stress state.

2a) Why are most crystalline materials linear elastic?

b) Why are rubbers almost incompressible?

c) Why does the Young's modulus of a crystalline material decrease with increasing temperature while the Young's modulus of a rubber increases with increasing temperature?
\[ E_1 = E_2 = 5 \text{ GPa} \quad \nu_{12} = 0.25 \quad G_{13} = 8 \text{ GPa}. \]
\[ E_3 = 25 \text{ GPa} \quad \nu_{13} = 0.33. \]

\[
\sigma_{ij} = \begin{bmatrix} 10 & 0 & 8 \\ 0 & 5 & 3 \\ 8 & 3 & 5 \end{bmatrix} \text{MPa}.
\]

\[ U = ? \]

\[ U = \frac{1}{2} \left[ \sigma_1 E_1 + \sigma_2 E_2 + \sigma_3 E_3 + \sigma_4 E_4 + \sigma_5 E_5 + \sigma_6 E_6 \right]. \]

**NEED COMPLIANCE MATRIX TO OBTAIN STRAINS**

\[ S_{11} = S_{22} = \frac{1}{E_1} = 0.2 \text{ GPa}^{-1} \]
\[ S_{33} = \frac{1}{E_3} = 0.04 \text{ GPa}^{-1} \]
\[ S_{44} = \frac{1}{G_{23}} = \frac{1}{6} \frac{1}{E_1} = \frac{1}{8} = 0.125 \text{ GPa}^{-1} \]
\[ S_{55} = \frac{1}{G_{31}} = \frac{1}{8} = 0.125 \text{ GPa}^{-1} \]
\[ S_{66} = 0 \quad \text{DONT NEED S_{66}.} \]

**APPLY \( \sigma_1 \) ONLY**

\[ \nu_{12} = \frac{E_2}{E_1} = \frac{-S_{12}}{S_{11}} \quad \frac{S_{12}}{E_1} = \frac{-0.25}{5} = -0.05 \]

\[ S_{12} = -0.05 \]

**APPLY \( \sigma_3 \) ONLY**

\[ \nu_{13} = \frac{E_3}{E_1} = \frac{-S_{13}}{S_{11}} \quad \frac{S_{13}}{E_1} = \frac{-0.33}{5} = -0.066 \]

\[ S_{13} = -0.066 \]

\[ S_{22} = S_{13} = -0.066 \]
\[ \varepsilon_1 = S_{11} \sigma_1 + S_{12} \sigma_2 + S_{13} \sigma_3 = (0.2) (0.010) + (-0.05) (0.005) + (-0.066) (0.05) = 0.00142 \]

\[ \varepsilon_2 = S_{22} \sigma_1 + S_{23} \sigma_2 + S_{32} \sigma_3 = (-0.05) (0.010) + (0.2) (0.005) + (-0.066) (0.05) = 0.00017 \]

\[ \varepsilon_3 = S_{33} \sigma_1 + S_{32} \sigma_2 + S_{23} \sigma_3 = (-0.066) (0.010) + (-0.066) (0.005) + (0.04) (0.05) = -0.00079 \]

\[ \varepsilon_4 = S_{44} \sigma_4 = (0.125) (0.023) = 0.00288 \]

\[ \varepsilon_5 = S_{55} \sigma_5 = (0.125) (0.001) = 0.00125 \]

\[ \varepsilon_6 = 0 \]

Strain Energy \[ = \frac{1}{2} \left( \sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3 + \sigma_4 \varepsilon_4 + \sigma_5 \varepsilon_5 + \sigma_6 \varepsilon_6 \right) \varepsilon_6 \]

\[ = \frac{1}{2} \left[ (0.2)(0.00142) + (0.2)(0.00017) + (0.04)(0.00079) + (0)(0.00288) + (0)(0.00125) \right] \]

\[ = 0.01012 \text{ MPa} = 0.01012 \text{ MN/m}^2 = 0.01012 \text{ MJ/m}^3 \]

\[ = 10.1 \text{ kJ/m}^3 \]

\# 2 (a) \[ E = \frac{1}{2} \left[ \frac{d^2 u}{dr^2} + T \frac{d^2 s}{dr^2} \right] \]

=0 crystalline materials - little change in entropy as strain \[ \frac{d s}{dr} \rightarrow 0 \]

=0 energy separation curve \[ u \]

=0 derivative of energy-separation curve at small strain \[ F = \frac{du}{dr} \]

=0 linear \[ \sigma \propto \varepsilon \]
2(b) Rubbers almost incompressible.

- Hydrostatic pressure - structure undergoes uniform volumetric compression
- Relative position of atoms in the chains remains similar
- To deform rubber hydrostatically, have to deform C-C bonds
- \( K \propto \frac{d^2 u_a}{de^2} \)

- Under tension or compression or shear, leading cause structure to become more ordered \( \Rightarrow \) large change in entropy without deforming C-C bonds \( \Rightarrow E, G \propto \frac{d^2 s}{de^2} \)

- Relatively easy to slide chains over one another compared to stretching or compressing C-C bonds

- \( K \gg E \)

- \( \nu = \frac{1}{2} - \frac{1}{6} \frac{E}{K} \Rightarrow 0 \Rightarrow \frac{1}{2} \Rightarrow \) material is incompressible.

2(c) Crystalline materials \( \Rightarrow E \propto \frac{d^3 U}{de^2} \); the second derivative of \( U \) with respect to distance decreases slightly as \( r \) increases. On heating, \( r \) increases \( \Rightarrow E \).

Rubbers: on heating, rubber has more thermal energy which increases the tendency of the rubber to go back to its random configuration.

- \( E = E^R \)

- \( E \propto -T \frac{d^2 G}{de^2}; E = 2n_r kT \) as \( T \uparrow \) \( E^R \)
3.225 Electronic and Mechanical Properties of Materials

Mechanical Properties: Test 2

Friday July 26, 2002

1a) At a temperature of 50°C, an amorphous polystyrene (T_g = 40°C) has a 10 minute relaxation modulus of 0.05 GPa. How long does it take to reach the same value of the relaxation modulus at a temperature of 60°C? (10 points)

Time-temperature shift constants:

<table>
<thead>
<tr>
<th>T_0 (°K)</th>
<th>C_1</th>
<th>C_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>T_g</td>
<td>-17.44</td>
<td>51.6</td>
</tr>
<tr>
<td>T_g+50</td>
<td>-8.86</td>
<td>101.6</td>
</tr>
</tbody>
</table>

1b) Spring-dashpot models can be used to represent the linear viscoelastic behaviour of polymers. The simplest such models are the Maxwell and Voigt models with the spring and dashpot in series and parallel, respectively. Describe the limitations of the Voigt spring-dashpot model. (5 points)

2a) An aluminum alloy (σ_y = 170MPa) component is subjected to the following stress state:

\[
\sigma_{ij} = \begin{bmatrix}
10n & 3n & 2n \\
3n & 5n & n \\
2n & n & 10n
\end{bmatrix} \text{ MPa}
\]

What value of n is sufficient to initiate yield in the component? (5 points)

2b) Why is the intrinsic lattice resistance of covalently bonded solids much larger than that of metals? (5 points)

2c) Describe two methods of alloying metals to increase their yield strength. Develop equations describing the increase in yield strength. (10 points)
3a) Creep data are given in the attached tables. Calculate the steady state creep strain rates for 316 stainless steel with a grain size of 100μm at a stress of 150 MPa and a temperature of 1000°C for diffusional flow and power law creep. The atomic volume for 316 SS is 1.21 x 10^{-29} m³. Note that Boltzmann’s constant is k = 1.38 x 10^{-23} J/K and the gas constant R = 8.314 J/mole/°K. (10 points)

3b) The two main mechanisms of diffusional flow are vacancy diffusion and grain boundary diffusion. Why is the activation energy for vacancy diffusion higher than that for grain boundary diffusion? (5 points)

4 A double edge notch specimen of PMMA is loaded in tension. The width, W of the specimen is 30mm, the thickness B is 5 mm and the edge notch length a is 3mm. The fracture toughness of the PMMA is 0.5 MPa m^{1/2} and the yield strength is 75 MPa. The geometrical factor Y for the stress intensity factor calculation is 1.12 for a double edge notch specimen.

4 a) What stress is required to break the specimen? (4 points)

4 b) What is the plane strain plastic zone size for this material just prior to fracture? (3 points)

4 c) Does the test specimen meet the requirements of a plane strain fracture toughness test? (3 points)

(60 points total)
<table>
<thead>
<tr>
<th>Material</th>
<th>Table 5.6</th>
<th>Table 6.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brandywine</td>
<td>6.7</td>
<td>5.7</td>
</tr>
<tr>
<td>Calypso</td>
<td>6.8</td>
<td>6.2</td>
</tr>
<tr>
<td>New Rambler</td>
<td>6.9</td>
<td>6.3</td>
</tr>
<tr>
<td>Russian Blue</td>
<td>7.0</td>
<td>7.4</td>
</tr>
<tr>
<td>Scarlet Rose</td>
<td>7.1</td>
<td>7.5</td>
</tr>
</tbody>
</table>

### Table 5.6: Lattice Diffusion Coefficients

<table>
<thead>
<tr>
<th>Material</th>
<th>V</th>
<th>V/1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brandywine</td>
<td>12</td>
<td>12.000</td>
</tr>
<tr>
<td>Calypso</td>
<td>13</td>
<td>13.000</td>
</tr>
<tr>
<td>New Rambler</td>
<td>14</td>
<td>14.000</td>
</tr>
<tr>
<td>Russian Blue</td>
<td>15</td>
<td>15.000</td>
</tr>
<tr>
<td>Scarlet Rose</td>
<td>16</td>
<td>16.000</td>
</tr>
</tbody>
</table>

### Table 6.2: Diffusion Coefficient Parameters

<table>
<thead>
<tr>
<th>Material</th>
<th>V</th>
<th>V/1000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brandywine</td>
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<tr>
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<td>18.000</td>
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<tr>
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<td>19</td>
<td>19.000</td>
</tr>
<tr>
<td>Russian Blue</td>
<td>20</td>
<td>20.000</td>
</tr>
<tr>
<td>Scarlet Rose</td>
<td>21</td>
<td>21.000</td>
</tr>
<tr>
<td>Material</td>
<td>0.0 FPY</td>
<td>0.1 FPY</td>
</tr>
<tr>
<td>----------</td>
<td>--------</td>
<td>--------</td>
</tr>
<tr>
<td>Steel</td>
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</tr>
<tr>
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<td>Copper</td>
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</tr>
<tr>
<td>Nickel</td>
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<tr>
<td>Copper-Nickel</td>
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<tr>
<td>Stainless Steel</td>
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<td></td>
</tr>
<tr>
<td>Nickel</td>
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<td></td>
</tr>
<tr>
<td>Titanium</td>
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</tr>
<tr>
<td>Titanium Alloys</td>
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<tr>
<td>Titanium</td>
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<td>1.3</td>
</tr>
<tr>
<td>Tantalum</td>
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<td>1.7</td>
</tr>
<tr>
<td>Tungsten</td>
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<tr>
<td>Tantalum Carbide</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tantalum Carbide</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
1. (a) \[ \log \alpha_T = \frac{C_1(T - T_0)}{C_2 + T - T_0} = \log \frac{t_T}{t_{T_0}} \]

\[ T_g = 40^\circ C = T_0 \]

\[ T_1 = 50^\circ C, \quad E_r(10\text{min}) = 0.05 \text{GPa} \]

\[-D \quad T_g = T_0 \quad C_1 = -17.44 \quad C_2 = 51.6 \]

\[ \log \frac{t_{50^\circ C}}{t_{40^\circ C}} = \frac{-17.44(50 - 40)}{51.6 + 50 - 40} = -2.83 \]

\[ \frac{t_{50^\circ C}}{t_{40^\circ C}} = 10^{-2.83} \]

5

\[ t_{40^\circ C} = \frac{t_{50^\circ C}}{10^{-2.83}} = 10\text{min} \times 10^{2.83} = 6780 \text{min} \]

\[ \log \frac{t_{60^\circ C}}{t_{40^\circ C}} = \frac{-17.44(60 - 40)}{51.6 + 60 - 40} = -4.87 \]

5

\[ \frac{t_{60^\circ C}}{t_{40^\circ C}} = 10^{-4.87} \]

\[ t_{60^\circ C} = (10^{-4.87})(6780 \text{min}) = 0.09 \text{ min} \times 60\text{sec/m}\text{in} = 5.5\text{sec} \]

(b) Voigt element limitations

- creep response - no initial elastic strain
- no recoverable strain
- stress relaxation response - stress is constant with time, rather than decaying.

5
\[ \sigma_{ij} = \begin{bmatrix} 10n & 3n & 2n \\ 3n & 5n & n \\ 2n & n & 10n \end{bmatrix} \text{ MPa} \quad \sigma_y = 170 \text{ MPa} \]

n for yield?

Van Huis: \[ \sigma_y = \frac{1}{2} \left[ \left( \sigma_{11} - \sigma_{22} \right)^2 + \left( \sigma_{22} - \sigma_{33} \right)^2 + \left( \sigma_{33} - \sigma_{11} \right)^2 \right] + 3\sigma_{12}^2 + 3\sigma_{23}^2 + 3\sigma_{31}^2 \]

\[ = \sqrt{\frac{1}{2} \left[ 25n^2 + 25n^2 + 0 \right] + (3)(9n^2) + 3(4n^2) + 3n^2} \]

\[ = \sqrt{25n^2 + 27n^2 + 12n^2 + 3n^2} \]

\[ = 6.18n \]

\[ n = \frac{170 \text{ MPa}}{8.18 \text{ GPa}} = 20.8 \]

2. (a)

Intrinsic lattice resistance covalently bonded materials > metals

- dislocations break bonds - at a time, yield related to \( Q_b = \frac{2U_b}{z} \)
- binding energy = 2 contributions from all atoms; similar for both coval. + metal.
- coval. solids - only nearest neighbors contribute \( (2n+2) \)
- metals - free e⁻; many layers at neighbors contribute \( (L\rightarrow\infty) \)
- \( Q_b \) for metals \( \approx Q_b \) for ceramics
- intrinsic lattice resistance found by equating work done by dislocation breaking 1 bond with \( Q_b = \sigma \Delta \alpha \approx 2 \left( \rho \sigma \sigma \right) \)

3. (c) allaying techniques

2x5 copy lecture notes
316 stainless

$T = 1000^\circ C$

$T = 1273^\circ K$

$\sigma_0 = 83.5 \text{ MPa}$

$n = 7.9$

$K = 8.314 \text{ J/mole/}^\circ \text{K}$

$\dot{\varepsilon} = \dot{\varepsilon}_0 \left( \frac{\sigma}{\sigma_0} \right)^n = A \exp \left( -\frac{Q}{RT} \right) \left( \frac{T}{T_0} \right)^n$

$= (1 \text{ sec}) \exp \left( -\frac{270000 \text{ J/mole}}{(8.314) (1273^\circ K) (5/\text{mole/}^\circ \text{K})} \right) \left( \frac{150}{33.5} \right)^{7.9}$

$= (8.33 \times 10^{-12}) (139 \times 10^{-5})$

$= 1.15 \times 10^{-6} /\text{sec}$

Diff. flow

$\dot{\varepsilon} = \frac{2\sigma \nu}{kTd^2} \left( \frac{D_v}{D_0} + \frac{2.5\delta}{d} \right)$

$D_v = D_0 \exp \left( -\frac{Q}{RT} \right)$

$= 3.7 \times 10^{-5} \exp \left( -\frac{289000}{(8.314) (1273)} \right)$

$= 1.20 \times 10^{-16} \text{ m}^2/\text{s}$

$\delta D_0 = 2.0 \times 10^{-13} \exp \left( -\frac{167000}{(8.314) (1273)} \right)$

$= 2.81 \times 10^{-10} \text{ m}^3/\text{sec}$
3b) • A is the activation energy for atom to jump between sites in the lattice.

• Vacancy diffusion = need energy to form vacancy, $U_f$
  \[ Q_v = Q + U_f \]

5 • grain boundary diffusion = gbd - more open structure
  - easier for diffusion to occur
  - $Q_b \approx 0.6 \times Q$

• \[ q_b < Q_v. \]