3.225: Electrical and Mechanical Properties of Materials
Test - 1
Tuesday, October 23, 2007
2:30 PM – 4:00 PM

Problem 1.
(a) For an anisotropic material, prove that

i. compliance matrix is symmetric (i.e., $S_{ij} = S_{ji}$)

ii. $\nu_{12}E_2 = \nu_{21}E_1$

(b) Derive the expressions of the elastic constant $S_{11}, S_{22}, S_{33}, S_{12}, S_{13}$ and $S_{23}$ in terms of the properties $E_1, E_2, E_3, \nu_{21}, \nu_{31}$ and $\nu_{32}$ for an anisotropic material.

(c) An anisotropic material is stamped out in a rigid cylindrical die as shown in the picture below. Assume the die walls are frictionless, and that the anisotropic material is in an elastic stress state.

Applied normal stress is $\sigma$ (i.e., $\sigma_1 = \sigma$). The elastic constants of the material give a hydrostatic stress state in the specimen. Prove that

i. $\frac{E_2}{E_3} = \frac{1 - \nu_{21}}{\nu_{32}}$

ii. At what applied stress, $\sigma$, the material would start yielding (use von Mises criterion)?

Problem 2.
A beam-like laminate with two isotropic layers of equal thickness is subjected to a temperature increase. The top layer has a Young’s modulus $E_1$ and a coefficient of thermal expansion $\alpha_1$. The bottom layer has a Young’s modulus $E_2 = 0.2E_1$ and a coefficient of thermal expansion $\alpha_2 = 10\alpha_1$. 
(a) Calculate the minimum and maximum stress in each layer in terms of the radius of curvature, \( \rho \), \( E_1 \) and \( \alpha_1 \).

(b) Sketch your result, showing the stress profile through the thickness of the laminate.

(c) Calculate the ratio of yield strengths of the two layers (i.e., \( \sigma_{y2}/\sigma_{y1} \)) in order for both the layers to yield at the same increase in temperature.

**Problem 3.**
(a) For an amorphous polymer, describe the changes with temperature in the relaxation modulus at a constant time \( (E_r(t)) \). Also plot \( E_r \) vs. \( T \) showing the changes in \( E_r \) for an amorphous polymer and a crystalline polymer on the same plot.

(b) The creep compliance, \( J(t) \) of a polymer is represented as: \( J(t) = (a\sqrt{t} + b) \). The applied stress is represented in figure below. The developed strains, \( \epsilon(t) \), at two times \( t_1 \) and \( t_2 \) are represented as below. Find out the constants ‘a’ and ‘b’.

![Diagram showing stress profile and strains](image)

\[
\begin{align*}
\Delta \sigma & \quad \Delta \sigma \\
\tau & \quad 3\tau \\
\begin{array}{c}
t_1 = 2\tau \\
\epsilon(t_1) = \epsilon
\end{array} & \quad \begin{array}{c}
t_2 = 7\tau \\
\epsilon(t_2) = 2.5\epsilon
\end{array}
\end{align*}
\]

(c) What are the limitations of Maxwell and Voigt models?

**Problem 4.**
(a) Plot Tresca and von Mises yield boundaries in 2-dimensions. Write the equations for the boundaries. Assume that \( \sigma_2 = 0 \) and the axis labels are in terms of \( (\sigma_1/\sigma_y) \) and \( (\sigma_3/\sigma_y) \).

(b) For a material, the yield strength becomes 5/6 times that of the intrinsic lattice resistance at a temperature \( T \) and a strain rate \( \dot{\gamma} \). Find the ratio of the plastic zone size of the same material, \( r_p \), at two different temperatures \( T \) and \( 1.2T \) for the same strain rate, \( \dot{\gamma} \). Assume that \( Q_b \) for the material is \( \approx 60 \) KT. Explain ductile-brittle transition phenomena from this.

- Good luck 😊 -