18.01 Exam 2

Name: __________________________________________

Problem 1 _______ /20
Problem 2 _______ /40
Problem 3 _______ /30
Problem 4 _______ /10

Total: __________ /100

Instructions: Please write your name at the top of every page of the exam. The exam is closed book, calculators are not allowed, but you are allowed to use your prepared index card. You will have approximately 50 minutes for this exam. The point value of each problem is written next to the problem – use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., raise your hand.
Problem 1 (20 points) Compute the following derivatives. Show all work, or you will not receive credit.

(a) (10 points) 
\[
\frac{d}{d\theta} \left( \frac{2\tan(\theta)}{1 - (\tan(\theta))^2} \right)
\]

(b) (10 points) 
\[
\frac{d}{dt} \sqrt{1 - (\sin(2t))^2}, \quad -\pi/4 < t < \pi/4.
\]
Problem 2 (40 points) For $x > 0$, the function $f(x)$ is defined by,

$$f(x) = \sqrt{x} + \frac{1}{\sqrt{x}}.$$

For the purposes of this problem, $\sqrt{2} \approx 1.4$, $\sqrt{3} \approx 1.7$ and $\sqrt{5} \approx 2.2$. Be sure you work with the correct function. If you work with the wrong function, few points will be given.

(a) (3 points) Write the equation of each vertical asymptote. If none exist, write “none exist”.

(b) (3 points) Write the equation of each horizontal asymptote. If none exist, write “none exist”.

(c) (7 points) On the number line, identify where $f'(x)$ is positive, negative or zero.
(d) (3 points) Write the coordinates of each local maximum. If none exist, write “none exist”.

(e) (3 points) Write the coordinates of each local minimum. If none exist, write “none exist”.

(f) (4 points) Write the coordinates of each inflection point. If none exist, write “none exist”.

(g) (7 points) On the number line, identify where \( f''(x) \) is positive, negative or zero.
(h) (10 points) On the grid given, sketch the graph of $y = f(x)$. 
Problem 3 (30 points) A box is made from two identical square sheets of metal with edge length $E$. A small square of edge length $x$ is removed and the two flaps are folded up. Find the value of $x$ that maximizes the volume of the box.
Problem 4 (10 points) Find the quadratic approximation of $\sqrt{2 - \cos(\theta)}$ for $\theta \approx 0$. 