

MIT OpenCourseWare
<http://ocw.mit.edu>

18.01 Single Variable Calculus
Fall 2006

For information about citing these materials or our Terms of Use, visit: <http://ocw.mit.edu/terms>.

18.01 Exam 3

Problem 1. (20 pts) Evaluate the following integrals

$$a) \int_0^2 \frac{xdx}{(1+x^2)^2}$$

$$b) \int_{-\pi/2}^{\pi/2} \sin^6 x \cos x dx$$

Problem 2. (20 pts.) Find the following approximations to

$$\int_0^{\pi/2} \cos x dx$$

(Do not give a numerical approximation to square roots; leave them alone.)

- Using the upper Riemann sum with two intervals
- Using the trapezoidal rule with two intervals
- Using Simpson's rule with two intervals

Problem 3. (20 points) Find the volume of the solid of revolution formed by revolving the y -axis the region enclosed by

$$y = \cos(x^2)$$

and the x -axis (central hump, only).

Problem 4. (20 points) Students studying for an exam get x hours of sleep in the two days leading up to the exam, where x is the range $0 \leq x \leq a$. The numbers of students who got between x_1 and x_2 hours of sleep is given by

$$\int_{x_1}^{x_2} cx dx; 0 \leq x_1 \leq x_2 \leq a$$

- What fraction of the student got less than $a/2$ hours of sleep?
- Their scores are proportional to the amount of sleep they got: $S(x) = 100(x/a)$. Find the (correctly weighted) average score in the class.

Problem 5. (20 points) Let

$$F(x) = \int_0^x \sqrt{t} \sin t dt$$

- a) Find $F'(x)$ for $x > 0$ identify the points $a > 0$ $F'(a) = 0$
- b) Decide whether F has a local maximum at the smallest critical point $a > 0$ that you found in part (a) by evaluating F'' .
- c) Say whether $F(x)$ is positive, negative or zero at each of the following points, and give a reason in each case.
 - i) $x=0$
 - ii) $x=\pi$
 - iii) $x=2\pi$
- d) Use a change of the variable to express $G(x) = \int_0^x u^2 \sin(u^2) du$ in terms of F .