18.01 Practice Exam 2

Problem 1. (20) Find the local maxima and minima and points of inflection of

$$2x^3 + 3x^2 - 12x + 1 \ .$$

Then use this data to sketch its graph on the given axes, showing also where it is convex (concave up) or concave (down). (Note that different scales are used on the two axes.)

Problem 2. (20) A new junk food — NoKarb PopKorn — is to be sold in large cylindrical metal cans with a removable plastic lid instead of a metal top. The metal side and bottom will be of uniform thickness, and the volume is fixed to be 64π cubic inches.

What base radius r and height h for the can will require the least amount of metal?

Show work, and include an argument to show your values for r and h really give a minimum.

Problem 3. (15) Evaluate the following indefinite integrals:

a) $\displaystyle\int e^{-3x}\,dx$

b) $\displaystyle\int \cos^2 x \sin x\,dx$

c) $\displaystyle\int \frac{x\,dx}{\sqrt{1-x^2}}$

Problem 4. (15)

A searchlight L is 100 meters from a prison wall. It is rotating at a constant rate of one revolution every 8 minutes. (How many radians/minute is that?)

Martha, an escaping prisoner, is running along the wall trying to keep just ahead of the beam of light. At the moment when the searchlight angle θ is 60 degrees, how fast does she have to run?

Problem 5. (15: 5, 10)

a) What value for the constant c will make the function $e^{-x}\sqrt{1+cx}$ approximately constant, for values of x near 0? (Show work.)

b) Find the solution $x(t)$ to the differential equation $\dfrac{dx}{dt} = 2t\sqrt{1-x^2}$ which also satisfies the condition $x(0) = 1$.

Problem 6. (15: 8,7)

a) Use the Mean-value Theorem to show that $\ln(1+x) < x$, if $x > 0$.

(You do not have to state the theorem.)

b) Let c be any constant. Show that the function $f(x) = x^3 + x + c$ cannot have two zeros.

(Use the Mean-value theorem, or some other argument.)