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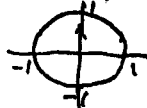
18.01 Single Variable Calculus
Fall 2006

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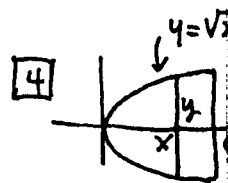
180) Practice EXAM 3 Sol'ns Fall 2006

1) a) $\cos 2x = \cos^2 x - \sin^2 x$
 $= (1 - \sin^2 x) - \sin^2 x$
 $= 1 - 2\sin^2 x$
 $\int \sin^2 x dx = \int \frac{1 - \cos 2x}{2} dx$
 $= \frac{x}{2} - \frac{\sin 2x}{4} + c$

b) $D(x \ln x) = \ln x + x \cdot \frac{1}{x}$
 $= \ln x + 1$
 \therefore by the fundamental theorem,
 $x \ln x \Big|_1^e = \int_1^e \ln x dx + \int_1^e 1 dx$
 $e - 1 - 0 = \int_1^e \ln x dx + e - 1$
 $\therefore \int_1^e \ln x dx = 1$

2) By horizontal slices, (calculate vol. of top half + double it)

 $= \int_0^1 \pi x^2 dy = \int_0^1 \pi(1-y^2) dy$
 $= \pi(y - \frac{y^3}{3}) \Big|_0^1 = \pi \cdot \frac{2}{3}$
 By cylindrical shells: $y = (1-x^2)^{1/4}$
 $= \int_0^1 2\pi x \cdot (1-x^2)^{1/4} dx$
 $= -\frac{4\pi}{5} (1-x^2)^{5/4} \Big|_0^1 = \frac{4\pi}{5}$
 \therefore Volume is $\frac{8\pi}{5} \approx \frac{8 \cdot (3.14)}{5} > \frac{25}{5}$
 5 cubic feet is not enough.

3) a) $F(x) = \int_0^x t^2 e^{-t^2} dt$; $F'(x) = x^2 e^{-x^2}$ (second fund thm)
 b) $F' = 0$ when $x=0$; otherwise $F'(x) > 0$. Thus F is increasing, so $x=0$ is a point of horiz. inflection (not a max or min)
 c) $u = t^2$; $\int_0^9 \sqrt{u} e^{-u} du = \int_0^3 t e^{-t^2} \cdot 2t dt = 2 \cdot F(3)$
 d) $e^{-t^2} \leq 1$
 $\therefore \int_0^x t^2 e^{-t^2} dt \leq \int_0^x t^2 dt = \frac{x^3}{3}$

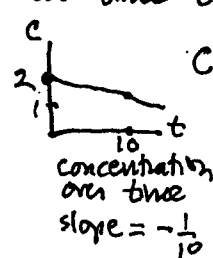
4) 
 Area of slice at x is $\pi y^2 = \pi x$
 Average area of slices $= \frac{1}{a} \int_0^a \pi x dx = \frac{1}{a} \pi \frac{x^2}{2} \Big|_0^a = \frac{1}{2} \pi x$

Therefore average area $= \frac{\pi a}{2}$
 which is the area of the slice at $x_0 = a/2$ (halfway)
 $\pi \cdot (\sqrt{x_0})^2 = \frac{\pi a}{2} \Rightarrow x_0 = a/2$

5)

1	8	15	22	29
3	2	0	1	3

a) by trapezoidal rule:
 Total # hits $\approx (\frac{3}{2} + 2 + 0 + 1 + \frac{3}{2}) \cdot 7 = 6 \cdot 7 = 42$
 b) by Simpson's rule:
 Total # hits $\approx (\frac{3 + 4 \cdot 2 + 2 \cdot 0 + 4 \cdot 1 + 3}{6}) \cdot 14 = \frac{18}{6} \cdot 14 = 42$

6) In an infinitesimal time interval dt at time t ,

 $c = 2 - \frac{1}{10}t$
 flow rate = $t^2(10-t)^2 \cdot 10^4$ (pool surface is $(100 \text{ cm})^2$)
 \therefore amt entering from t to $t+dt$
 $= t^2(10-t)^2 \cdot 10^4 \cdot (2 - \frac{t}{10}) \cdot dt$
 Total amt = $10^4 \int_0^{10} t^2(10-t)^2(2 - \frac{t}{10}) dt$ nanograms
 For Δt calculation: replace dt by Δt in $\textcircled{6}$
 write $\textcircled{6}$ as $\sum_1^n 10^4 t_i^2 (10-t_i)^2 (2 - t_i/10) \Delta t$
 and pass to limit as $n \rightarrow \infty$ integral given.