18.01 Single Variable Calculus
Fall 2006

18.01 Practice Questions for Exam 4 – Fall 2006

Problem 1. Evaluate $\displaystyle\int \frac{x-4}{(x+1)(x^2+4)}\,dx$.

Problem 2. Evaluate $\displaystyle\int_0^2 \frac{dx}{(x^2+4)^2}$ by making the substitution $x = 2\tan u$.

Problem 3.

a) Derive a reduction formula relating $\displaystyle\int_0^1 x^{2n}e^{-x^2}dx$ to $\displaystyle\int_0^1 x^{2n-2}e^{-x^2}dx$.

b) Let $F(x) = \displaystyle\int_0^1 e^{-x^2}dx$. Express $\displaystyle\int_0^1 x^2e^{-x^2}dx$ in terms of values of $F(x)$.

Problem 4. Find the volume of the solid obtained by rotating about the y-axis the finite region bounded by the positive x- and y-axes and the graph of $y = \cos x$.

Problem 5. Make a reasonable sketch of one loop of the polar curve $r = \sin 3\theta$, and find the area inside it.

Problem 6. Let $x(t) = \cos^3 t$, $y(t) = \sin^3 t$, $0 \le t \le \pi/2$ be a parametric representation of a curve.

a) Compute the arclength of the curve.

b) Compute the surface area of the surface formed by rotating the curve around the x-axis.

Problem 7. Set up an integral for the length of one arch of the curve $y = \sin x$, and by estimating the integral, tell how this length compares with $\pi\sqrt{2}$.

Problem 8. A circular metal disc of radius a has a non-constant density δ (units: gms/cm^2); the density at a point P on the disc is given by $\delta = r^2$, where r is the distance of the point from the center of the disc. Set up and evaluate a definite integral giving the total mass of the disc.

Problem 9.

a) Sketch the curve given in polar coordinates by $r = 1 + \cos\theta$

b) Find the polar coordinates of the following two points (show work):

(i) where the curve in part (a) intersects the circle of radius 3/2 centered at the origin;

(ii) where the above curve intersects the circle of radius 3/2 centered at the point $x = 3/2$ on the x-axis.

Other kinds of problems:

Other kinds of partial fractions decompositions;
sketching curves given parametrically, finding their arclength;
finding surface area for rotated curves in xy-coordinates;
deriving polar equations of curves given geometrically, changing from rectangular equations to polar and vice-versa.