EXAM 1

(1) (10 points) Find \( \int_{-2}^{3} 2x^2 \lfloor |x| \rfloor \, dx \). (Here, as usual, \([x]\) denotes the largest integer \( \leq x \).)

(2) (10 points) Let \( f \) be an integrable function on \([a, b]\) and \( a < d < b \). Further suppose that

\[
\int_{a+d}^{b+d} f(x - d) \, dx = 4, \quad \int_{-a}^{-d} f(-x) \, dx = 7.
\]

Find

\[
\int_{d}^{b} 2f(x) \, dx.
\]
(3) (10 points) Suppose $A, B$ are inductive sets. Prove $A \cap B$ is an inductive set. Give an example of inductive sets $A, B$ such that $A - B$ is not an inductive set.

(4) (15 points) Let $f$ be a bounded, integrable function on $[0,1]$. Suppose there exists $C \in \mathbb{R}$ such that $f(x) \geq C > 0$ for all $x \in [0,1]$. Prove that $g(x) = 1/f(x)$ is integrable on $[0,1]$. 
(5) (15 points) Suppose $f$ is defined for all $x \in (-1, 1)$ and that $\lim_{x \to 0} f(x) = A$. Show there exists a constant $c < 1$ such that $f(x)$ is bounded for all $x \in (-c, c)$. 