(1) (10 points) Evaluate
\[
\lim_{x \to 0} \left( \frac{1}{x} - \frac{1}{\log(x + 1)} \right)
\]

(2) (10 points) Evaluate
\[
\int \frac{3x - 2}{x^2 - 6x + 10} \, dx
\]
(3) (10 points) Let $f$ be an infinitely differentiable function on $\mathbb{R}$. We say $f$ is analytic on $(-1,1)$ if the sequence $\{T_n f(x)\}$ converges to $f(x)$ for all $x \in (-1,1)$, where $T_n f(x)$ is the $n$th Taylor polynomial of $f$ centered at zero. Suppose there exists a constant $0 < C \leq 1$ such that
$$|f^{(k)}(x)| \leq C^k k!$$
for every positive integer $k$ and every real number $x \in (-1,1)$. Prove that $f$ is analytic on $(-1,1)$.

(4) (10 points) Let $f(x)$ be a function defined on $(0, \pi]$. Suppose $\lim_{n \to \infty} f(1/n) = 0$ and $\lim_{n \to \infty} f(\pi/n) = 1$. Prove that $\lim_{x \to 0^+} f(x)$ does not exist.
(5) A function $f$ on $\mathbb{R}$ is compactly supported if there exists a constant $B > 0$ such that $f(x) = 0$ if $|x| \geq B$. If $f$ and $g$ are two differentiable, compactly supported functions on $\mathbb{R}$, then we define

$$(f * g)(x) = \int_{-\infty}^{\infty} f(x - y)g(y)dy.$$ 

Note: We define $\int_{-\infty}^{\infty} f(x)dx = \lim_{t \to \infty} \int_{-t}^{0} f(x)dx + \lim_{t \to \infty} \int_{0}^{t} f(x)dx$.

- (10 points) Prove $(f * g)(x) = (g * f)(x)$.

- (10 points) Prove $(f' * g)(x) = (g' * f)(x)$. 
18.014 Calculus with Theory
Fall 2010

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