(1) Evaluate \( \int \frac{t^3 + t}{\sqrt{1+t^2}} \, dt \)

(2) Evaluate \( \int_{3}^{5} x^3 \sqrt{x^2 - 9} \, dx \)

(3) Suppose that \( \lim_{x \to a^+} g(x) = B \neq 0 \) where \( B \) is finite and \( \lim_{x \to a^+} h(x) = 0 \), but \( h(x) \neq 0 \) in a neighborhood of \( a \). Prove that

\[
\lim_{x \to a^+} \left| \frac{g(x)}{h(x)} \right| = \infty.
\]

(4) Let \( f(x) : [0, \infty) \to \mathbb{R}^+ \) be a positive continuous function such that \( \lim_{x \to \infty} f(x) = 0 \). Prove there exists \( M \in \mathbb{R}^+ \) such that \( \max_{x \in [0, \infty)} f(x) = M \).

(5)  

- A sequence is called \textit{Cauchy} if for all \( \epsilon > 0 \) there exists \( N \in \mathbb{Z}^+ \) such that for all \( m, n > N \), \( |a_m - a_n| < \epsilon \). Prove that if \( \{a_n\} \) is a convergent sequence, then it is Cauchy. (The converse is also true.)

- A function \( f : \mathbb{R} \to \mathbb{R} \) is called a \textit{contraction} if there exists \( 0 \leq \alpha < 1 \) such that \( |f(x) - f(y)| \leq \alpha |x - y| \). Let \( f \) be a contraction. For any \( x \in \mathbb{R} \), prove the sequence \( \{f^n(x)\} \) is Cauchy, where \( f^n(x) = f \circ f \circ \cdots \circ f(x) \) (the \( n \) times composition of \( f \) with itself).