Derivative of $\ln(\sec x)$

Now let’s use the chain rule to take the derivative of $\ln(\sec x)$.

\[
\frac{d}{dx}(\ln(\sec x)) = \frac{(\sec x)'}{\sec x} = \frac{\sec x \tan x}{\sec x} = \tan x
\]

Oddly enough, this strange looking function is not only interesting as a review of the chain rule. The natural log was invented before the exponential function by a man named Napier, exactly in order to evaluate functions like this.

People cared about these functions a lot because they were used in navigation. In order to quickly and accurately multiply sines and cosines together for navigation, Napier used a logarithm. Logarithms were invented long before people knew about exponents, and it was a surprise when it was discovered that they were connected to exponents. The natural log was invented before the log base ten and everything else, exactly for this kind of purpose.