Quotient Rule Practice

Find the derivatives of the following rational functions.

a) \( \frac{x^2}{x+1} \)

b) \( \frac{x^4+1}{x^2} \)

c) \( \frac{\sin(x)}{x} \)

Solution

a) \( \frac{x^2}{x+1} \)

The quotient rule tells us that if \( u(x) \) and \( v(x) \) are differentiable functions, and \( v(x) \) is non-zero, then:

\[
\left( \frac{u(x)}{v(x)} \right)' = \frac{u'(x)v(x) - u(x)v'(x)}{(v(x))^2}
\]

In this problem \( u = x^2 \) and \( v = x + 1 \), so \( u' = 2x \) and \( v' = 1 \). Applying the quotient rule, we see that:

\[
\left( \frac{x^2}{x+1} \right)' = \frac{2x \cdot (x+1) - x^2 \cdot 1}{(x+1)^2}
= \frac{2x^2 + 2x - x^2}{(x+1)^2}
= \frac{x^2 + 2x}{(x+1)^2}.
\]

b) \( \frac{x^4+1}{x^2} \)

Here \( u(x) = x^4 + 1 \), \( u'(x) = 4x^3 \), \( v(x) = x^2 \) and \( v'(x) = 2x \). The quotient rule tells us that:

\[
\left( \frac{x^4+1}{x^2} \right)' = \frac{4x^3 \cdot x^2 - (x^4 + 1) \cdot 2x}{(x^2)^2}
= \frac{4x^5 - 2x^5 - 2x}{x^4}
= \frac{2x^4 - 2}{x^3}.
\]
c) $\frac{\sin(x)}{x}$

The derivative of $\sin(x)$ is $\cos(x)$ and the derivative of $x$ is 1, so the quotient rule tells us that:

$$\left( \frac{\sin(x)}{x} \right)' = \frac{(\cos(x)) \cdot x - (\sin(x)) \cdot 1}{x^2} = \frac{x \cos(x) - \sin(x)}{x^2}.$$  

When we learn to evaluate this expression at $x = 0$, it will tell us that the slope of the graph of $\text{sinc}(x) = \frac{\sin(x)}{x}$ is 0 when $x = 0$.  
