Removable Discontinuities

At a **removable** discontinuity, the left-hand and right-hand limits are equal but either the function is not defined or not equal to these limits:

\[
\lim_{x \to x_0^+} f(x) = \lim_{x \to x_0^-} f(x) = f(x_0)
\]

![Figure 1: A removable discontinuity: the function is continuous everywhere except one point](image)

For example, \( g(x) = \frac{\sin(x)}{x} \) and \( h(x) = \frac{1 - \cos x}{x} \) are defined for \( x \neq 0 \), but both functions have removable discontinuities. This is not obvious at all, but we will learn later that:

\[
\lim_{x \to 0} \frac{\sin x}{x} = 1 \quad \text{and} \quad \lim_{x \to 0} \frac{1 - \cos x}{x} = 0.
\]

So both of these functions have removable discontinuities at \( x = 0 \) despite the fact that the fractions defining them have a denominator of 0 when \( x = 0 \).