How to Check Your Answer

While it may be difficult to solve a differential equation, it is fairly easy to see if a proposed solution is correct. Check the following results by plugging the proposed answer into the original equation.

a) \( y = \frac{1}{3}e^x \) is a solution to \( 4y'' - y = e^x \).

b) \( y = \frac{1}{x} \) is a solution to \( x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = 0 \).

Solution

a) \( y = \frac{1}{3}e^x \) is a solution to \( 4y'' - y = e^x \).

If \( y = \frac{1}{3}e^x \) then \( y' = \frac{1}{3}e^x \) and \( y'' = \frac{1}{3}e^x \). We now plug these expressions in to the original equation:

\[
4y'' - y = 4 \left( \frac{1}{3}e^x \right) - \frac{1}{3}e^x = \frac{3}{3}e^x = e^x.
\]

It is true that \( 4y'' - y = e^x \) when \( y = \frac{1}{3}e^x \).

b) \( y = \frac{1}{x} \) is a solution to \( x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = 0 \).

Here \( y = x^{-1} \), \( \frac{dy}{dx} = -x^{-2} \) and \( \frac{d^2y}{dx^2} = 2x^{-3} \). Plugging in to the original equation we get:

\[
x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = x^2 \cdot 2x^{-3} + 3x \cdot (-x^{-2}) + x^{-1} = 2x^{-1} - 3x^{-1} + x^{-1} = 0.
\]

Therefore \( y = \frac{1}{x} \) is a solution to the differential equation \( x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = 0 \).