Definition of the Definite Integral

The definite integral $\int_a^b f(x)dx$ describes the area “under” the graph of $f(x)$ on the interval $a < x < b$.

![Figure 1: Area under a curve](image)

Abstractly, the way we compute this area is to divide it up into rectangles then take a limit. The three steps in this process are:

1. Divide the region into “rectangles”
2. Add up areas of rectangles
3. Take the limit as the rectangles become infinitesimally thin

Figure 2 shows the area under a curve divided into rectangles. Notice that since the rectangles aren’t curved they do not exactly overlap the area. Adding up the areas of the rectangles doesn’t give you exactly the area under the curve, but the two areas are pretty close together.

The key idea is that as the rectangles get thinner, the difference between the area covered by the rectangles and the area under the curve will get smaller. In the limit, the area covered by the rectangles will exactly equal the area under the curve.
Figure 2: Area under a curve divided into rectangles