Riemann Sums

We haven’t yet finished with approximating the area under a curve using sums of areas of rectangles, but we won’t use any more elaborate geometric arguments to compute those sums.

\[ y = f(x) \]

Figure 1: Area under a curve

The general procedure for computing the definite integral \( \int_a^b f(x) \, dx \) is:

- Divide \([a, b]\) into \(n\) equal pieces of length \( \Delta x = \frac{b - a}{n} \).
- Pick any value \(c_i\) in the \(i^{th}\) interval and use \(f(c_i)\) as the height of the rectangle.
- Sum the areas of the rectangles:

\[
(f(c_1)\Delta x) + (f(c_2)\Delta x) + \cdots + (f(c_n)\Delta x) = \sum_{i=1}^{n} f(c_i)\Delta x
\]

The sum \( \sum_{i=1}^{n} f(c_i)\Delta x \) is called a Riemann Sum.

This notation is supposed to be reminiscent of Leibnitz’ notation. In the limit as \(n\) goes to infinity, this sum approaches the value of the definite integral:

\[
\lim_{n \to \infty} \sum_{i=1}^{n} f(c_i)\Delta x = \int_a^b f(x) \, dx
\]
Which is the area under the curve $y = f(x)$ above $[a, b]$. 