Riemann Sum Practice

Use a Riemann sum with \( n = 6 \) subdivisions to estimate the value of \( \int_0^2 (3x + 2) \, dx \).

Solution

This solution was calculated using the left Riemann sum, in which \( c_i = x_{i-1} \) is the left endpoint of each of the subintervals of \([a, b]\). To denote the heights of the rectangles we let \( y_i = f(x_i) \), and so obtain the following expression for the left Riemann sum:

\[
f(c_1)\Delta x + f(c_2)\Delta x + \cdots + f(c_n)\Delta x = (y_0 + y_1 + \ldots + y_{n-1})\Delta x.
\]

In our example, \( n = 6 \), \( a = 0 \), \( b = 2 \), \( \Delta x = \frac{b-a}{n} = \frac{1}{3} \), and the values \( y_i \) correspond to the height of the graph of \( y = 3x + 2 \) at the left edge of each interval, as illustrated in Figure 1.

![Figure 1: Rectangles used to compute the Riemann sum.](image)

We could compute \( x_i = a + i\Delta x = \frac{i}{3} \) and so \( y_i = 3(x_i) + 2 = i + 2 \) and \( c_i = \frac{i-1}{3} \), or we could simply mark off the left endpoints \( 0, \frac{1}{3}, \frac{2}{3}, 1, \frac{4}{3}, \frac{5}{3} \) and \( \frac{7}{3} \) and then read the heights of the rectangles from the graph. In either case, our formula for the left Riemann sum tells us that the area under the graph of \( 3x + 2 \) between \( a = 0 \) and \( b = 2 \) is approximately:

\[
(y_0 + y_1 + \ldots + y_{n-1})\Delta x = (2 + 3 + 4 + 5 + 6 + 7) \cdot \frac{1}{3} = 9.
\]
Because \( \int_0^2 (3x + 2) \, dx \) is the area of a trapezoid with width 2 and sides of height 2 and 8, we can easily check our work:

\[
\int_0^2 (3x + 2) \, dx = 2 \cdot \frac{2 + 8}{2} = 10.
\]

From the figure we see that the left Riemann sum slightly underestimates the area, so our answer of 9 is probably correct.