

The Fundamental Theorem and Negative Integrands

We said that the fundamental theorem would tell us the distance traveled if we knew the speed we were traveling at every instant. But if we make a round trip, the difference $x(b) - x(a)$ is 0. We'd like the fundamental theorem to notice whether our velocity was in the positive or negative direction and cancel the change in position when appropriate.

Goal: Extend integration to the case $f(x) < 0$.

Although we assumed that the graph of the function was above the x axis when we originally described definite integrals, it turns out that we don't have to change our notation to accommodate functions with negative outputs.

Example: $\int_0^{2\pi} \sin(x) dx$

Let's try evaluating the "area under" two humps of the graph of the sine function.

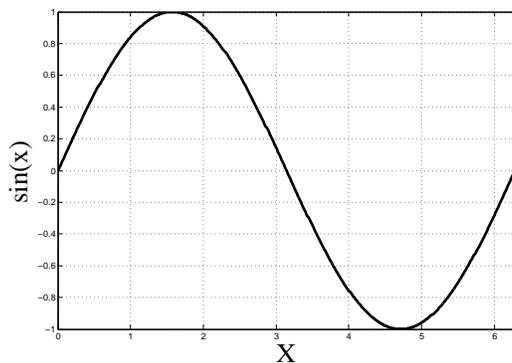


Figure 1: $\sin(x)$ for $0 < x < 2\pi$

$$\int_0^{2\pi} \sin x dx = (-\cos x)|_0^{2\pi} = -\cos(2\pi) - (-\cos(0)) = -1 - (-1) = 0$$

If we're going to insist that the fundamental theorem of calculus must be true even when $f(x) < 0$, then the definite integral is *not* exactly the area between the curve and the x -axis. The definite integral does equal the area under the curve when the graph is above the x -axis, but when the graph is below the x -axis the value we get from the fundamental theorem is negative.

The true geometric interpretation of the definite integral is that it adds up the area above the x -axis (and below the graph of the function) and subtracts the area below the x -axis (above the graph of the function).

Question: Wouldn't you use the absolute value of the velocity?

Answer: You would use the absolute value of the velocity to compute the *total* distance traveled. Without the absolute value, the definite integral measures the *net* distance traveled.

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