Warning about units.

Previously, we calculated the volume of a parabolic “cauldron” to be $\frac{\pi}{2}a^2$. There’s something fishy about this expression — it looks as if it has units of area, but it’s describing a volume. In general, we must be very aware of what units we’re using.

Suppose the height of the cauldron is $a = 100\text{cm}$. Then:

$$V = \frac{\pi}{2}(100)^2 \text{cm}^3$$
$$= \frac{\pi}{2}10^4 \text{cm}^3$$
$$= \frac{\pi}{2}10 \sim 16 \text{ liters}$$

Next, suppose that the height of the cauldron is $a = 1\text{m}$. Then:

$$V = \frac{\pi}{2}(1)^2 \text{m}^3$$
$$= \frac{\pi}{2}10^6 \text{cm}^3$$
$$= \frac{\pi}{2}1000 \sim 1600 \text{ liters}$$

But $100\text{cm} = 1\text{m}$. Why are the answers different?

The problem is that we don’t know the units in the equation $y = x^2$. If the units are centimeters, then $100\text{cm} = 10^2\text{cm}$. If the units are meters then $1\text{m} = 1^2\text{m}$. When we use centimeters as units, the cauldron is five times as tall as it is wide, so it looks like:

![Figure 1: Cauldron cross section for units of centimeters.](image)

When we interpret $y = x^2$ in meters, we find that the cauldron is twice as wide as it is tall, which seems more likely in the context of the problem.

This confusion about units arose because the equation $y = x^2$ is not scale-invariant.