Average Bank Balance

An amount of money $A_0$ compounded continuously at interest rate $r$ increases according to the law:

$$ A(t) = A_0 e^{rt} \quad (t=\text{time in years}) $$

a) What is the average amount of money in the bank over the course of $T$ years?

b) Check your work by plugging in $A_0 = 100$, $r = .05$ and $T = 1$; does the result seem plausible?

Solution

a) What is the average amount of money in the bank over the course of $T$ years?

The average value of a function $f(x)$ over the interval $[a, b]$ is:

$$ \frac{1}{b-a} \int_a^b f(x) \, dx. $$

In our example, the function is $A(t) = A_0 e^{rt}$ and the interval is $[0, T]$. Noting that the antiderivative of $e^{rt}$ is $\frac{1}{r} e^{rt}$, we find:

$$ \text{Avg}(A) = \frac{1}{T-0} \int_0^T A_0 e^{rt} \, dt $$

$$ = A_0 \frac{1}{rT} e^{rt} \bigg|_0^T $$

$$ = A_0 \frac{1}{rT} (e^{rT} - e^{0}) $$

$$ \text{Avg}(A) = \frac{A_0}{rT} (e^{rT} - 1). $$

This is the difference between the final and initial balance, divided by rate times time!

b) Check your work by plugging in $A_0 = 100$, $r = .05$ and $T = 1$; does the result seem plausible?

If $r = .05$ and $T = 1$ then:

$$ \text{Avg}(A) = \frac{100}{.05} (e^{.05} - 1) $$

$$ = $102.5 $$

This seems plausible because if we were dealing with simple interest (rather than continuously compounded interest), our starting balance would be $100 and our final balance would be $105.