Weighted Averages

A *weighted average* is calculated by dividing the weighted total value of a fraction by the total of the weighting function:

$$\frac{\int_a^b f(x)w(x) \, dx}{\int_a^b w(x) \, dx}.$$  

Multiplying by $w(x)$ makes some values of $f(x)$ contribute more to the total than other values, depending on the value of $x$ and $w(x)$. Dividing by the integral of $w(x)$ is analogous to dividing by the length or by the number of values.

First we check that this makes sense by confirming that the weighted average of a constant is that same constant:

$$\frac{\int_a^b cw(x) \, dx}{\int_a^b w(x) \, dx} = \frac{c \int_a^b w(x) \, dx}{\int_a^b w(x) \, dx} = c.$$  

We see that we were correct to put $\int_a^b w(x) \, dx$ in the denominator.

Now pretend you have a stock which you bought for $10 one year. Six months later you brought some more for $20, and then you bought some more for $30. What’s the average price of your stock?

It depends on how many shares you bought. If you bought $w_1$ shares the first time, $w_2$ shares the second time and $w_3$ shares the third time, the total amount that you spent is

$$10w_1 + 20w_2 + 30w_3.$$  

The average price per share is the total price divided by the total number of shares:

$$\frac{10w_1 + 20w_2 + 30w_3}{w_1 + w_2 + w_3}.$$  

This is the discrete analog of the continuous average

$$\frac{\int_a^b f(x)w(x) \, dx}{\int_a^b w(x) \, dx}.$$  

The function $f$ is the function describing the price of a share and the weights are the amounts (relative importance) of the different purchases.

**Question:** You can’t factor out the $f(x)$, can you?

**Answer:** When we found the weighted average of a constant, we factored out $c$. In

$$\frac{\int_a^b f(x)w(x) \, dx}{\int_a^b w(x) \, dx}$$

we cannot factor out $f(x)$. If the weighted average is interesting you have to do two different integrals to calculate it. It’s only when $f(x)$ is constant that you can factor it out (in which case, the calculation is not very interesting at all).