Another Reduction Formula: \( \int x^n e^x \, dx \)

To compute \( \int x^n e^x \, dx \) we derive another reduction formula. We could replace \( e^x \) by \( \cos x \) or \( \sin x \) in this integral and the process would be very similar.

Again we'll use integration by parts to find a reduction formula. Here we choose \( u = x^n \) because

\[ u' = nx^{n-1} \]

is a simpler (lower degree) function. If \( u = x^n \) then we'll have to have

\[ v' = e^x, \quad v = e^x. \]

(Note that the antiderivative of \( v \) is no more complicated than \( v' \) was — another indication that we've chosen correctly.)

On the other hand, if we used \( u = e^x \), then \( u' = e^x \) would not be any simpler.

Performing the integration by parts we get:

\[
\int x^n e^x \, dx = \frac{x^n e^x}{uv'} - \int \frac{x^{n-1} e^x}{u'v} \, dx.
\]

If:

\[ G_n(x) = \int x^n e^x \, dx \]

then we get the reduction formula:

\[ G_n(x) = x^n e^x - nG_{n-1}(x). \]

Let’s illustrate this by computing a few integrals. First we directly compute:

\[ G_0(x) = \int x^0 e^x \, dx = e^x + c. \]

Now we can use the reduction formula to conclude that:

\[ G_1(x) = xe^x - G_0(x) = xe^x - e^x + c. \]

So \( \int x e^x \, dx = xe^x - e^x + c. \)

**Question:** How do you know when this method will work?

**Answer:** Good question! The answer is “only through experience and practice”. To use this method on an integrand, we need one factor \( u \) of the integrand to get simpler when we differentiate and the other factor \( v \) not to get more complicated when we integrate.

We’ve seen how to use integration by parts to derive reduction formulas. We could also find these formulas by advanced guessing — guess what the formula should be and then check it. Either method is valid.