Translating $y = 1$ into Polar Coordinates

We’ll take a simple description from rectangular coordinates, $y = 1$, and translate it into polar coordinates. To do this, we plug in the (definitive) formula $y = r \sin \theta$.

$$
y = r \sin \theta \\
1 = r \sin \theta \\
r = \frac{1}{\sin \theta}
$$

In rectangular coordinates the line has equation $y = 1$. In polar coordinates its equation is $r = \frac{1}{\sin \theta}$.

![Figure 1: $r = \frac{1}{\sin \theta}$](image)

As indicated in Figure 1, for different values of $\theta$ points on the horizontal line are different distances $r$ from the origin. That distance $r$ is $\frac{1}{\sin \theta}$.

We need one more piece of information to complete this problem; what is the range of $\theta$? When $\theta = 0$ the denominator of the expression describing $r$ is 0; this corresponds to one end of the line. As $\theta$ increases from 0 to $\pi$, $r$ decreases to 1 at $\theta = \frac{\pi}{2}$ and then increases to infinity again.

Our final answer is:

$$
r = \frac{1}{\sin \theta}, \quad 0 < \theta < \pi.
$$

**Question:** Is it typical to express $r$ as a function of $\theta$? Does it matter?

**Answer:** In this course our answers will almost always describe $r$ as a function of $\theta$, but it’s not required. We do it this way because we like:

$$
r = \frac{1}{\sin \theta}
$$

better than:

$$
\theta = \sin^{-1} \left( \frac{1}{r} \right).
$$