

**PROFESSOR:** Welcome back to recitation. In this video I want us to look at the following problem. We're going to let  $f$  of  $x$  equal  $1$  over  $x$ . And I want to consider the solid generated by rotating  $f$  of  $x$  about the  $x$ -axis between  $x$  equal  $1$  and  $x$  equal infinity. I want you to find the area of a cross-sectional slice, and then I want you to find the volume of the solid. In the cross-sectional slice, the one I'm interested in is the one that you see in the  $xy$ -plane.

So my suggestion to you is that right away draw yourself a picture, just at least a rough sketch of the curve  $y$  equals  $f$  of  $x$ , and then what that cross-sectional slice will look like and start off from there. And take a while to work on that and then I'll be back and show you what I did.

OK welcome back. Well, hopefully you were able to make some headway on this problem, and maybe you found some interesting things. Hopefully you did. And we will see what I find and if they are interesting, which I think they are.

So let's start off. I told you the first thing you should do is get a rough picture. So I'm going to draw a rough picture and make sure that those match up well, that my picture matches up with yours. So let me give you the  $xy$ -plane. And then  $y$  equals  $1$  over  $x$  is a curve that at  $x$  equal  $1$  - I'm going to make  $y$  equal  $1$  up here.  $y$  equal  $1$  up there.  $x$  equal  $1$ . So when  $x$  is  $1$ ,  $y$  is  $1$ . And then it's going to decay down. As  $x$  goes to infinity it decays

So it looks something like that. And then I actually should have moved this  $1$  because I'm going to need that side to get my cross-sectional slice. So my cross sectional slice is going to come down here and look something like this. Oops, that's supposed to be symmetric. That's still supposed to be something like this. It's supposed to be symmetric about the  $x$ -axis. And the cross-sectional slice is the area of that.

Now, if you remember what you saw in lecture. You saw, I believe, that the integral from  $1$  to infinity of  $1$  over  $x$   $dx$  diverges, because you end up with  $\log$  evaluated at infinity and that is infinite and so you wind up with this. This actually diverges. That was the area of the top part from  $0$  to  $f$  of  $x$ . So of course if I double that, I'm still going to get that it diverges. So in fact the cross sectional slice area, CSS area, is infinite.

OK, so that's one part. The other part is to find the volume of the solid of revolution. So the second part was take this piece from  $0$  up to  $f$  of  $x$ , rotate it around the  $x$ -axis and compute the volume. Now we know how to compute the volume. This is the disc method that I'm going to

use here. And I will write out what we need to do and then we'll look at what the integral gives us.

So for the disc method, so let's say this is the volume part. So the volume, it's going to be the disc method. So I need to do  $\pi r^2 dx$ . And in this case I'm integrating from 1 to infinity, so I need  $\pi r^2 dx$ . I'm going to be integrating something that's  $\pi r^2 dx$ . And I know the bounds are 1 to infinity, and so I need  $r$  in terms of, as a function of  $x$ . That's fairly easy.  $r$  is just the measure from 0 up to  $f$  of  $x$ . So that's just  $1/x$ . So this is actually the integral-- I'm going to pull the  $\pi$  out-- of  $1/x^2$  from 1 to infinity  $dx$ .

OK so how do I do this integral? Well you were shown in class that in actuality this is the limit, as some value up here goes to infinity, of this integral, but we're just going to do it the shorthand way that he also mentioned in class and keep the infinity around. So we can keep that as our bounds, knowing we're taking limits as this thing goes to infinity.

So the integral of  $1/x^2$  is  $-1/x$ . So I get  $-\pi$  times  $1/x$ , evaluated at 1 in infinity. I can just check that if I need to, but this is  $x^{-1}$  and its derivative is  $-x^{-2}$ . So I needed that negative there. And then I evaluate this. Well at infinity,  $1/x$ -- as  $x$  goes to infinity, this first part goes to 0. So this is 0 minus  $-\pi$  times  $1/1$ . So  $-\pi$  times 1. So this is just  $\pi$ .

OK, so hopefully that kind of blows your mind a little bit, that you could have something where this cross-sectional slice is infinite, but in fact if you look at it the way we computed the volume, we have in fact a finite volume. So I don't know what else I'm going to say about that except I think it's really cool. And you can think about why that is.

And in fact you might want to notice that we had to-- we computed things in terms of cross-sectional slices coming from the other direction. So we looked at these cross-sectional slices and we got our-- we showed our volume was finite there. So we had a sum of a bunch of finite things and so it made sense that you were going to get-- and the finite things were getting small fast enough. That's the point. That when you add up these finite things that are getting small fast enough, you can still end up with a finite number.

But I guess, yeah, that's where I'll stop. So just to say that we were looking at this sort of solid of revolution problem again, but we were dealing with improper integrals, and we were showing how you can do these kinds of problems with improper integrals. So now I really will stop there.

