

General Power Series

We now know exactly which geometric series converge and, if they do converge, what they converge to. Our next step is to extend this result to cover all power series. In general, a *power series* is a series of the form:

$$a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots = \sum_{n=0}^{\infty} a_nx^n.$$

As with geometric series, there's a very simple rule that tells us when power series converge. The series converges when $-R < x < R$ for some "magic number" R called the *radius of convergence*. The value of R depends on the values of the coefficients a_i . When $|x| > R$, the sum $\sum a_nx^n$ diverges. When $|x| = R$ the series might or might not converge — we won't deal with this case in this course.

What always happens is that for $|x| < R$, the values $|a_nx^n|$ tend to 0 exponentially fast. When $|x| > R$, the values $|a_nx^n|$ won't tend to 0 at all. For example, when $x = 1/2$ the geometric series looks like:

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$$

We're adding together numbers that get smaller and smaller. When $x = 2$ the numbers in our summation get bigger and bigger:

$$1 + 2 + 4 + 8 + \cdots$$

Question: How can you tell when the numbers a_nx^n are declining *exponentially* fast?

Answer: Any time the power series converges the numbers a_nx^n decline exponentially fast. This is because of the power x^n in each term.

Question: How do you find R ?

Answer: There's a long discussion of this question in many textbooks, but you won't need it. The radius of convergence will always be 1 or infinite or as obvious as it is for the geometric series.

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