Finding the Radius of Convergence

Use the ratio test to find the radius of convergence of the power series

\[ \sum_{n=1}^{\infty} \frac{x^n}{n}. \]

**Solution**

As Christine explained in recitation, to find the radius of convergence of a series \( \sum_{n=0}^{\infty} c_n x^n \) we apply the ratio test to find \( L = \lim_{n \to \infty} \left| \frac{c_{n+1} x^{n+1}}{c_n x^n} \right| \). The value of \( x \) for which \( L = 1 \) is the radius of convergence of the power series.

In this case,

\[ \frac{c_{n+1} x^{n+1}}{c_n x^n} = \frac{x^{n+1} (n + 1)}{x^n n} = x \cdot \frac{n}{n + 1}. \]

Taking the limit of this as \( n \) goes to infinity, we get:

\[ L = \lim_{n \to \infty} \left| x \cdot \frac{n}{n + 1} \right| = |x|. \]

When \( |x| < 1 \), \( L < 1 \) and the ratio test tells us that the series will converge. For \( |x| > 1 \), \( L > 1 \) and the series diverges. The radius of convergence is 1.

This gives us an idea of how close the harmonic series \( \sum_{n=1}^{\infty} \frac{1}{n} \) is to being convergent.