18.02 Multivariable Calculus
Fall 2007

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18.02 Practice Exam 2B

Problem 1. Let \( f(x, y) = x^2y^2 - x \).

a) (5) Find \( \nabla f \) at \((2, 1)\)

b) (5) Write the equation for the tangent plane to the graph of \( f \) at \((2, 1, 2)\).

c) (5) Use a linear approximation to find the approximate value of \( f(1.9, 1.1) \).

d) (5) Find the directional derivative of \( f \) at \((2, 1)\) in the direction of \(-\hat{i} + \hat{j}\).

Problem 2. (10) On the contour plot below, mark the portion of the level curve \( f = 2000 \) on which \( \frac{\partial f}{\partial y} \geq 0 \).

Problem 3. a) (10) Find the critical points of

\[ w = -3x^2 - 4xy - y^2 - 12y + 16x \]

and say what type each critical point is.

b) (10) Find the point of the first quadrant \( x \geq 0, y \geq 0 \) at which \( w \) is largest. Justify your answer.

Problem 4. Let \( u = y/x, v = x^2 + y^2, w = w(u, v) \).

a) (10) Express the partial derivatives \( w_x \) and \( w_y \) in terms of \( w_u \) and \( w_v \) (and \( x \) and \( y \)).

b) (7) Express \( xw_x + yw_y \) in terms of \( w_u \) and \( w_v \). Write the coefficients as functions of \( u \) and \( v \).

c) (3) Find \( xw_x + yw_y \) in case \( w = v^5 \).

Problem 5. a) (10) Find the Lagrange multiplier equations for the point of the surface

\[ x^4 + y^4 + z^4 + xy + yz + zx = 6 \]

at which \( x \) is largest. (Do not solve.)

b) (5) Given that \( x \) is largest at the point \((x_0, y_0, z_0)\), find the equation for the tangent plane to the surface at that point.

Problem 6. Suppose that \( x^2 + y^3 - z^4 = 1 \) and \( z^3 + zx + xy = 3 \).

a) (8) Take the total differential of each of these equations.

b) (7) The two surfaces in part (a) intersect in a curve along which \( y \) is a function of \( x \). Find \( dy/dx \) at \((x, y, z) = (1, 1, 1)\).