1. Let \((\bar{x}, \bar{y})\) be the center of mass of the triangle with vertices at \((-2, 0), (0, 1), (2, 0)\) and uniform density \(\delta = 1\).

a) (10) Write an integral formula for \(\bar{y}\). Do not evaluate the integral(s), but write explicitly the integrand and limits of integration.

b) (5) Find \(\bar{x}\).

2. (15) Find the polar moment of inertia of the unit disk with density equal to the distance from the \(y\)-axis.

3. Let \(\bar{F} = (ax^2y + y^3 + 1)i + (2x^3 + bxy^2 + 2)j\) be a vector field, where \(a\) and \(b\) are constants.

a) (5) Find the values of \(a\) and \(b\) for which \(\bar{F}\) is conservative.

b) (5) For these values of \(a\) and \(b\), find \(f(x, y)\) such that \(\bar{F} = \nabla f\).

c) (5) Still using the values of \(a\) and \(b\) from part (a), compute \(\int_C \bar{F} \cdot d\bar{r}\) along the curve \(C\) such that \(x = e^t \cos t, y = e^t \sin t, 0 \leq t \leq \pi\).

4. (10) For \(\bar{F} = yx^3i + y^2j\), find \(\int_C \bar{F} \cdot d\bar{r}\) on the portion of the curve \(y = x^2\) from \((0, 0)\) to \((1, 1)\).

5. Consider the region \(R\) in the first quadrant bounded by the curves \(y = x^2, y = x^2/5, xy = 2,\) and \(xy = 4\).

a) (10) Compute \(dxdy\) in terms of \(dudv\) if \(u = x^2/y\) and \(v = xy\).

b) (10) Find a double integral for the area of \(R\) in \(uv\) coordinates and evaluate it.

6. a) (5) Let \(C\) be a simple closed curve going counterclockwise around a region \(R\). Let \(M = M(x, y)\). Express \(\int_C Mdx\) as a double integral over \(R\).

b) (5) Find \(M\) so that \(\int_C Mdx\) is the mass of \(R\) with density \(\delta(x, y) = (x + y)^2\).

7. Consider the region \(R\) enclosed by the \(x\)-axis, \(x = 1\) and \(y = x^3\).

a) (5) Use the normal form of Green’s theorem to find the flux of \(\bar{F} = (1 + y^2)j\) out of \(R\).

b) (5) Find the flux out of \(R\) through the two sides \(C_1\) (the horizontal segment) and \(C_2\) (the vertical segment).

c) (5) Use parts (a) and (b) to find the flux out of the third side \(C_3\).