18.02 Practice Exam 3B

**Problem 1.** a) Draw a picture of the region of integration of \( \int_0^1 \int_x^{2x} dy \, dx \)

b) Exchange the order of integration to express the integral in part (a) in terms of integration in the order \( dx \, dy \). Warning: your answer will have two pieces.

**Problem 2.** a) Find the mass \( M \) of the upper half of the annulus, \( 1 < x^2 + y^2 < 9 \) (\( y \geq 0 \)) with density \( \delta = \frac{y}{x^2 + y^2} \).

b) Express the \( x \)-coordinate of the center of mass, \( \bar{x} \), as an iterated integral. (Write explicitly the integrand and limits of integration.) Without evaluating the integral, explain why \( \bar{x} = 0 \).

**Problem 3.** a) Show that \( \mathbf{F} = (3x^2 - 6y^2) \hat{i} + (-12xy + 4y) \hat{j} \) is conservative.

b) Find a potential function for \( \mathbf{F} \).

c) Let \( \mathbf{C} \) be the curve \( x = 1 + y^3(1 - y)^3, \ 0 \leq y \leq 1 \). Calculate \( \int_{\mathbf{C}} \mathbf{F} \cdot d\mathbf{r} \).

**Problem 4.** a) Express the work done by the force field \( \mathbf{F} = (5x + 3y) \hat{i} + (1 + \cos y) \hat{j} \) on a particle moving counterclockwise once around the unit circle centered at the origin in the form \( \int_a^b f(t) \, dt \). (Do not evaluate the integral; don’t even simplify \( f(t) \).)

b) Evaluate the line integral using Green’s theorem.

**Problem 5.** Consider the rectangle \( R \) with vertices \((0, 0), (1, 0), (1, 4)\) and \((0, 4)\). The boundary of \( R \) is the curve \( \mathbf{C} \), consisting of \( C_1 \), the segment from \((0, 0)\) to \((1, 0)\), \( C_2 \), the segment from \((1, 0)\) to \((1, 4)\), \( C_3 \) the segment from \((1, 4)\) to \((0, 4)\) and \( C_4 \) the segment from \((0, 4)\) to \((0, 0)\). Consider the vector field
\[
\mathbf{F} = (\cos x \sin y) \hat{i} + (xy + \sin x \cos y) \hat{j}
\]
a) Find the work of \( \mathbf{F} \) along the boundary \( \mathbf{C} \) oriented in a counterclockwise direction.

b) Is the total work along \( C_1, C_2 \) and \( C_3 \), more than, less than or equal to the work along \( C_4 \)?

**Problem 6.** Find the volume of the region enclosed by the plane \( z = 4 \) and the surface
\[
z = (2x - y)^2 + (x + y - 1)^2.
\]
Suggestion: change of variables.