Feel free to work with others, but the final write-up should be entirely your own and based on your own understanding.

1. (10 pts) Let $L$ be the line which passes through the point $(a, 0, 0)$ and is parallel to the $z$-axis. Let $D$ be the region that lies inside the cylinder of radius $a$ centred around the line $L$ and that lies between the planes $z = -1$ and $z = 3$. Describe the region $D$ in cylindrical coordinates.

2. (10 pts) Let $D$ be the region inside the sphere of radius $2a$ centred at the origin and that lies between the planes $x = -a$ and $x = a$. Describe the region $D$ in spherical coordinates.

3. (15 pts) Suppose that $f: A \rightarrow B$ and $g: B \rightarrow C$ are two functions, and let $g \circ f: A \rightarrow C$ be their composition. For each statement below, say whether the statement is true or false. If true, give a reason and if false give a counterexample.
   (i) If $f$ and $g$ are surjective, then $g \circ f: A \rightarrow C$ is surjective.
   (ii) If $g \circ f: A \rightarrow C$ is surjective, then $f$ is surjective.
   (iii) If $g \circ f: A \rightarrow C$ is surjective, then $g$ is surjective.

4. (10 pts) Let $S \subset \mathbb{R}^3$ be the right angled cone, with vertex at the origin and centred around the $z$-axis, and which lies on or above the $xy$-plane. Write down a function $f: \mathbb{R}^3 \rightarrow \mathbb{R}$ such that $S = f^{-1}(c)$ is the level set of $f$ at height $c$.

5. (10 pts) (2.1.34).
6. (5 pts) (2.2.9).
7. (5 pts) (2.2.11).
8. (5 pts) (2.2.13).
9. (5 pts) (2.2.15).
10. (5 pts) (2.2.31).
11. (5 pts) (2.2.35).
12. (5 pts) (2.2.42).

**Just for fun:** What is the volume of the intersection of three cylinders of radius $r$ and height $h$? Assume that the cylinders are centred around the three coordinate axes and that the central point of each cylinder is the origin.
18.022 Calculus of Several Variables
Fall 2010

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