1. 8.22: 14 (5 points)

2. 8.24: 12 (5 points)

3. Let \( f(x, y) = \int_0^y g(u)du \) where \( g : \mathbb{R} \rightarrow \mathbb{R} \) is a strictly positive continuous function.
   - Find \( \nabla f : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \) in terms of \( g \).
   - Consider a level set \( \{(x, y) \in \mathbb{R}^2 | f(x, y) = c\} \). Prove that for a fixed \( c \neq 0 \) there are exactly two level curves in the set. Moreover, prove they are precisely the graph of the function \( h(x) = b/x \) for exactly one \( b \in \mathbb{R} \). (Do not try to determine \( b \) in terms of \( g \)! Just prove it exists and is unique!)
   - Parameterize one curve on a level set and prove that \( \nabla f \) is orthogonal to the level set at each point on the curve.

   (6 points)

4. Let \( f : \mathbb{R}^2 \rightarrow \mathbb{R} \) by

   \[
   f(x, y) = \begin{cases} 
   xy \frac{x^2-y^2}{x^2+y^2} & (x, y) \neq (0, 0) \\
   0 & (x, y) = (0, 0)
   \end{cases}
   \]

   - Prove \( \frac{\partial f}{\partial x}(0, y) = -y \) for any \( y \) and \( \frac{\partial f}{\partial y}(x, 0) = x \) for any \( x \).
   - Prove \( \frac{\partial^2 f}{\partial y \partial x} \neq \frac{\partial^2 f}{\partial x \partial y} \).

   (6 points)

4. C20:5 (4 points)

5. C20:6 (4 points)