(1) (10 points) Consider the transformation \( T : \mathbb{R}^3 \to \mathbb{R}^3 \) such that \( T(1,0,0) = (2,1,4), T(0,1,0) = (4,3,6), T(0,0,1) = (0,-1,2) \).

(a) Determine the null space of \( T \).

(b) If \( A \) is the plane formed by \( \text{span}(\{(2,5,-3),(-1,-1,1)\}) \), write \( T(A) \) in parametric form.
(2) (10 points) Let

\[ F(t) = \begin{cases} 
  (\sin t, -\cos t) & t \in [0, \pi] \\
  (\sin t, \cos t + 2) & t \in (\pi, 2\pi]
\end{cases} \]

(a) Find \( F'(\pi) \), if it is well defined.
(b) Find \( F''(\pi) \), if it is well defined.
(c) Determine \( \kappa(t) \) everywhere it is defined.
(3) (10 points) Let \( f(x, y, z) = x^2 + y^2 + z^2 \). Prove \( f \) is differentiable at \( (1, 1, 1) \) with linear transformation \( T(x, y, z) = 2x + 2y + 2z \).
(4) (15 points) Consider the set $\mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$ of all linear maps $L$ from $\mathbb{R}^3$ to $\mathbb{R}^2$ and define addition of $L, K \in \mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$ the following way:

$$(L + K)(v) = L(v) + K(v) \quad (v \in \mathbb{R}^3)$$

Define multiplication by a constant $c$ as:

$$(cL)(v) = c(L(v)) \quad (v \in \mathbb{R}^3)$$

(a) Are the linear maps $L(x, y, z) = (x, 0), K(x, y, z) = (y, 0), N(x, y, z) = (x, y)$ linearly independent? Prove it either way.

(b) Find a basis for $\mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$.

(c) What is the dimension of $\mathcal{L}(\mathbb{R}^3, \mathbb{R}^2)$?
(5) (15 points) Consider the function $f: \mathbb{R}^2 \to \mathbb{R}$ that satisfies the following conditions:

(a) For all fixed $x_0 \in \mathbb{R}$ the function $f_{x_0} = f(x_0, y): \mathbb{R} \to \mathbb{R}$ is continuous and;

(b) For all fixed $y_0 \in \mathbb{R}$ the function $f_{y_0} = f(x, y_0): \mathbb{R} \to \mathbb{R}$ is continuous and;

(c) For all fixed $x_0 \in \mathbb{R}$ the function $f_{x_0}$ is monotonically increasing in $y$, i.e. if $y > y'$ then, $f(x_0, y) > f(x_0, y')$.

Prove $f$ is continuous.