NOTE: If at any point during a calculation you are using a theorem from class, justify the calculation by stating the appropriate theorem.

(1) (10 points) Consider \( f(x, y) = (xy + y)^{10} \) on the square \( Q = [0, 1] \times [0, 1] \). Evaluate \( \iint_{Q} f \, dx \, dy \).
(2) (5 points) Complete the following statement. (There is more than one correct answer.)
Let $S \subset \mathbb{R}^n$ be open and connected. Suppose $\mathbf{f}$ is a vector field defined on $S$. Then $\mathbf{f}$ is a gradient field if and only if $
abla u$, where $u$ is a scalar function such that $\mathbf{f} = \nabla u$.

(3) (10 points) Let $\gamma$ be the semi-circle connecting $(0,0)$ and $(2,0)$ that sits in the half plane where $y \geq 0$. Given $\mathbf{f}(x, y) = (2x \cos y - x \sin y + y^7)$, calculate $\int_{\gamma} \mathbf{f} \cdot d\gamma$. 


(4) Consider the surface $x^2yz + 2xz^2 = 6$ in $\mathbb{R}^3$.

(a) (3 points) For $(x, y) = (1, 4)$, determine all values of $z$ such that $(1, 4, z)$ is on the surface.

(b) (6 points) For each of the values of $z$ found above, determine at which of the points $(1, 4, z)$ one can find a neighborhood on the surface and a function $g : \mathbb{R}^2 \to \mathbb{R}$ such that the neighborhood can be described by the points $(x, y, g(x, y))$.

(c) (6 points) Choose one point from part (b) where the implicit function theorem can be applied and let $g(x, y) = z$ be the function defined in a neighborhood of $(1, 4)$ such that $(x, y, g(x, y))$ is on the surface. Find $\nabla g(1, 4)$. 

(5) (15 points) Assuming the comparison theorem for step functions, prove it for integrable functions $f, g : U \to \mathbb{R}$. That is, let $U$ be a closed rectangle in $\mathbb{R}^3$ and assume $\int_U f, \int_U g$ both exist. If $g \leq f$ for all $x \in U$, prove $\int_U g \leq \int_U f$. 
BONUS: (6 points)

(a) Let $A$ be a set of content zero and assume $B \subset A$. Prove $B$ has content zero.

(b) Let $A_i$, $i = 1, \ldots, n$ be sets of content zero. Prove $\bigcup_{i=1}^{n} A_i$ has content zero.

(c) Provide a counterexample to the following statement (and explain it):
Let $\{A_i\}_{i=1}^{\infty}$ be a collection of sets $A_i$ which each have content zero. Then $\bigcup_{i=1}^{\infty} A_i$ has content zero.