Equation of a plane

1. Find the equation of the plane containing the three points $P_1 = (1, 0, 1)$, $P_2 = (0, 1, 1)$, $P_3 = (1, 1, 0)$.

**Answer:** This problem is identical (with changed numbers) to the worked example we just saw.

The vectors $\overrightarrow{P_1P_2}$ and $\overrightarrow{P_1P_3}$ are in the plane, so

$$
N = \overrightarrow{P_1P_2} \times \overrightarrow{P_1P_3} = \begin{vmatrix}
i & j & k \\
-1 & 1 & 0 \\
0 & 1 & -1
\end{vmatrix} = i(-1) - j(1) + k(-1) = (-1, -1, -1).
$$

is orthogonal to the plane.

Now for any point $P = (x, y, z)$ in the plane, the vector $\overrightarrow{P_1P}$ is also in the plane and is therefore orthogonal to $N$. Expressing this with the dot product we get

$$
N \cdot \overrightarrow{P_1P} = 0
\iff (-1, -1, -1) \cdot (x - 1, y, z - 1) = 0
\iff -(x - 1) - y - (z - 1) = 0
\iff x + y + z = 2.
$$