Least Squares Interpolation

1. Use the method of least squares to fit a line to the four data points

\((0,0), \ (1,2), \ (2,1), \ (3,4)\).

**Answer:** We are looking for the line \(y = ax + b\) that best models the data. The deviation of a data point \((x_i, y_i)\) from the model is

\[y_i - (ax_i + b)\].

The sum of the squares of the deviation is

\[
D = (0 - (a \cdot 0 + b))^2 + (2 - (a \cdot 1 + b))^2 + (1 - (a \cdot 2 + b))^2 + (4 - (a \cdot 3 + b))^2
\]

\[= b^2 + (2 - a - b)^2 + (1 - 2a - b)^2 + (4 - 3a - b)^2.\]

Taking derivatives and setting them to 0 gives

\[
\frac{\partial D}{\partial a} = -2(2 - a - b) - 4(1 - 2a - b) - 6(4 - 3a - b) = 0 \Rightarrow 28a + 12b = 32 \Rightarrow 14a + 6b = 16
\]

\[
\frac{\partial D}{\partial b} = 2b - 2(2 - a - b) - 2(1 - 2a - b) - 2(4 - 3a - b) = 0 \Rightarrow 12a + 8b = 14 \Rightarrow 6a + 4b = 7.
\]

This linear system of two equations in two unknowns is easy to solve. We get

\[a = \frac{11}{10}, \ b = \frac{1}{10}.\]

Here is a plot of the problem.
18.02SC Multivariable Calculus
Fall 2010

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.