Chain Rule

1. The temperature on a hot surface is given by

\[ T = 100e^{-(x^2+y^2)}. \]

A bug follows the trajectory \( r(t) = (t \cos(2t), t \sin(2t)) \).

a) What is the rate that temperature is changing as the bug moves?

b) Draw the level curves of \( T \) and sketch the bug’s trajectory.

**Answer:**

a) The chain rule says

\[
\frac{dT}{dt} = \frac{\partial T}{\partial x} \frac{dx}{dt} + \frac{\partial T}{\partial y} \frac{dy}{dt}
\]

\[
= -200xe^{-(x^2+y^2)}(\cos(2t) - 2t \sin(2t)) - 200ye^{-(x^2+y^2)}(\sin(2t) + 2t \cos(2t)).
\]

You could stop here, or substitute \( x = t \cos(2t) \) and \( y = t \sin(2t) \). After simplification you get

\[
\frac{dT}{dt} = -200te^{-t^2}.
\]

b) The level curves of \( T \) are the curves \( x^2 + y^2 = \text{constant} \), i.e., circles. The bug moves in a spiral.

![Diagram of level curves of T](https://via.placeholder.com/150)

2. Suppose \( w = f(x, y) \) and \( x = t^2, y = t^3 \). Suppose also that at \((x, y) = (1, 1)\) we have \( \frac{\partial w}{\partial x} = 3 \) and \( \frac{\partial w}{\partial y} = 1 \). Compute \( \frac{dw}{dt} \) at \( t = 1 \).

**Answer:**

At \( t = 1 \) we have \( (x, y) = (1, 1) \), \( \frac{dx}{dt}\bigg|_{1} = 2 \), \( \frac{dy}{dt}\bigg|_{1} = 3 \). Therefore the chain rule says

\[
\frac{dw}{dt}\bigg|_{1} = \frac{\partial f}{\partial x}\bigg|_{(1,1)} \frac{dx}{dt}\bigg|_{1} + \frac{\partial f}{\partial y}\bigg|_{(1,1)} \frac{dy}{dt}\bigg|_{1} = 3(2) + 1(3) = 9.
\]