1. Let $(\bar{x}, \bar{y})$ be the center of mass of the triangle with vertices at $(-2, 0)$, $(0, 1)$, $(2, 0)$ and uniform density $\delta = 1$.
   a) (10) Write an integral formula for $\bar{y}$. Do not evaluate the integral(s), but write explicitly the integrand and limits of integration.
   b) (5) Find $\bar{x}$.

2. (15) Find the polar moment of inertia of the unit disk with density equal to the distance from the $y$-axis.

3. Let $\vec{F} = (ax^2y + y^3 + 1)i + (2x^3 + bxy^2 + 2)j$ be a vector field, where $a$ and $b$ are constants.
   a) (5) Find the values of $a$ and $b$ for which $\vec{F}$ is conservative.
   b) (5) For these values of $a$ and $b$, find $f(x, y)$ such that $\vec{F} = \nabla f$.
   c) (5) Still using the values of $a$ and $b$ from part (a), compute $\int_C \vec{F} \cdot d\vec{r}$ along the curve $C$ such that $x = e^t \cos t$, $y = e^t \sin t$, $0 \leq t \leq \pi$.

4. (10) For $\vec{F} = yx^3i + y^2j$, find $\int_C \vec{F} \cdot d\vec{r}$ on the portion of the curve $y = x^2$ from $(0, 0)$ to $(1, 1)$.

5. Consider the region $R$ in the first quadrant bounded by the curves $y = x^2$, $y = x^2/5$, $xy = 2$, and $xy = 4$.
   a) (10) Compute $dxdy$ in terms of $dudv$ if $u = x^2/y$ and $v = xy$.
   b) (10) Find a double integral for the area of $R$ in $uv$ coordinates and evaluate it.

6. a) (5) Let $C$ be a simple closed curve going counterclockwise around a region $R$. Let $M = M(x, y)$. Express $\int_C M dx$ as a double integral over $R$.
   b) (5) Find $M$ so that $\int_C M dx$ is the mass of $R$ with density $\delta(x, y) = (x + y)^2$.

7. Consider the region $R$ enclosed by the $x$-axis, $x = 1$ and $y = x^3$.
   a) (5) Use the normal form of Green’s theorem to find the flux of $\vec{F} = (1 + y^2)j$ out of $R$.
   b) (5) Find the flux out of $R$ through the two sides $C_1$ (the horizontal segment) and $C_2$ (the vertical segment).
   c) (5) Use parts (a) and (b) to find the flux out of the third side $C_3$. 

18.02 Practice Exam 3 A