Problems: Polar Coordinates and the Jacobian

1. Let $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1} \frac{y}{x}$. Directly calculate the Jacobian $\frac{\partial(r, \theta)}{\partial(x, y)} = \frac{1}{r}$.

2. For the change of variables $x = u$, $y = \sqrt{r^2 - u^2}$, write $dx\,dy$ in terms of $u$ and $r$. 