Problems: Work and Line Integrals

1. Evaluate \( I = \int_C y \, dx + (x + 2y) \, dy \) where \( C \) is the curve shown.

![Figure 1: Curve C is C_1 followed by C_2.](image)

**Answer:** The curve \( C \) is made up of two pieces, so

\[
I = \int_{C_1} y \, dx + (x + 2y) \, dy + \int_{C_2} y \, dx + (x + 2y) \, dy.
\]

Note that we don’t always need to introduce the variable \( t \).

\( C_1 : \ y = 1, \) use \( x \) as parameter. \( 0 \leq x \leq 1 \) \( \Rightarrow \ dx = dx, \ dy = 0. \)

\[
\Rightarrow \int_{C_1} y \, dx + (2 + 2y) \, dy = \int_0^1 dx = 1.
\]

\( C_2 : \ x = 1, \) use \( y \) as parameter. \( y \) goes from 1 to 0.

\[
\Rightarrow \int_{C_2} y \, dx + (2 + 2y) \, dy = \int_0^1 (1 + 2y) \, dy = -\int_0^1 1 + 2y \, dy = -2.
\]

So \( I = 1 - 2 = -1. \)

2. Let \( \mathbf{F} = -xi - yj. \) Sketch this vector field and describe it in words.

**Answer:** Each arrow starts at \((x, y)\) and ends at the origin. The further a vector in this field is from \((0,0)\), the longer it is.