Identifying Gradient Fields and Exact Differentials

1. Compute the curl of the tangential vector field $\mathbf{F} = \left\langle \frac{-y}{r^2}, \frac{x}{r^2} \right\rangle$.

2. Show that $\mathbf{F}$ is not conservative by computing $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $C$ is the unit circle.

3. Why do you think we refer to $\mathbf{F}$ as a “tangential” vector field?

4. In polar coordinates, $\theta(x, y) = \tan^{-1} \frac{y}{x}$. Show that $\mathbf{F} = \nabla \theta$. 