**Finding Area Using Line Integrals**

Use a line integral (and Green’s Theorem) to find the area of the unit circle.

**Answer:** Recall that Green’s Theorem tells us \( \oint_C M \, dx + N \, dy = \iint_R N_x - M_y \, dA \). To find the area of the unit circle we let \( M = 0 \) and \( N = x \) to get \( \iint_R 1 \, dA = \oint_C x \, dy \).

We parametrize the circle by \( x = \cos \theta, \ y = \sin \theta, \ 0 < \theta \leq 2\pi \), so \( x \, dy = \cos^2 \theta \, d\theta \). Then

\[
\text{Area} = \int \int_R 1 \, dA = \oint_C x \, dy = \int_0^{2\pi} \cos^2 \theta \, d\theta = \int_0^{2\pi} \frac{1 + \cos 2\theta}{2} \, d\theta = \left[ \frac{1}{2} \left( \theta + \frac{1}{2} \sin 2\theta \right) \right]_0^{2\pi} = \pi.
\]
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