Problems: Practice with Triple Integrals

Find the moment of inertia about the z-axis of a solid bounded by the paraboloid \( z = x^2 + y^2 \) and the plane \( z = 1 \). Assume the solid has uniform density 1.

**Answer:** We use the formula \( I = \iiint r^2 \rho \, dV \) with density \( \rho = 1 \). Converting to polar coordinates, the equation of the paraboloid becomes \( z = r^2 \) and we get the limits of integration \( 0 \leq r \leq \sqrt{z} \).

\[
I = \iiint_{\text{solid}} r^2 \rho \, dV
= \int_0^1 \int_0^{2\pi} \int_0^{\sqrt{z}} r^2 \cdot r \, dr \, d\theta \, dz
= \int_0^1 \int_0^{2\pi} \frac{z^2}{4} \, d\theta \, dz
= \int_0^1 \frac{\pi z^2}{2} \, dz
= \frac{\pi}{6}.
\]